TeVeS from Metric-Affine Gravity

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Outline

- **Metric-Affine Gravity**
- **Tensor-Vector Theory**
- **TeVeS from Metric-Affine Gravity**
- **Conclusion**

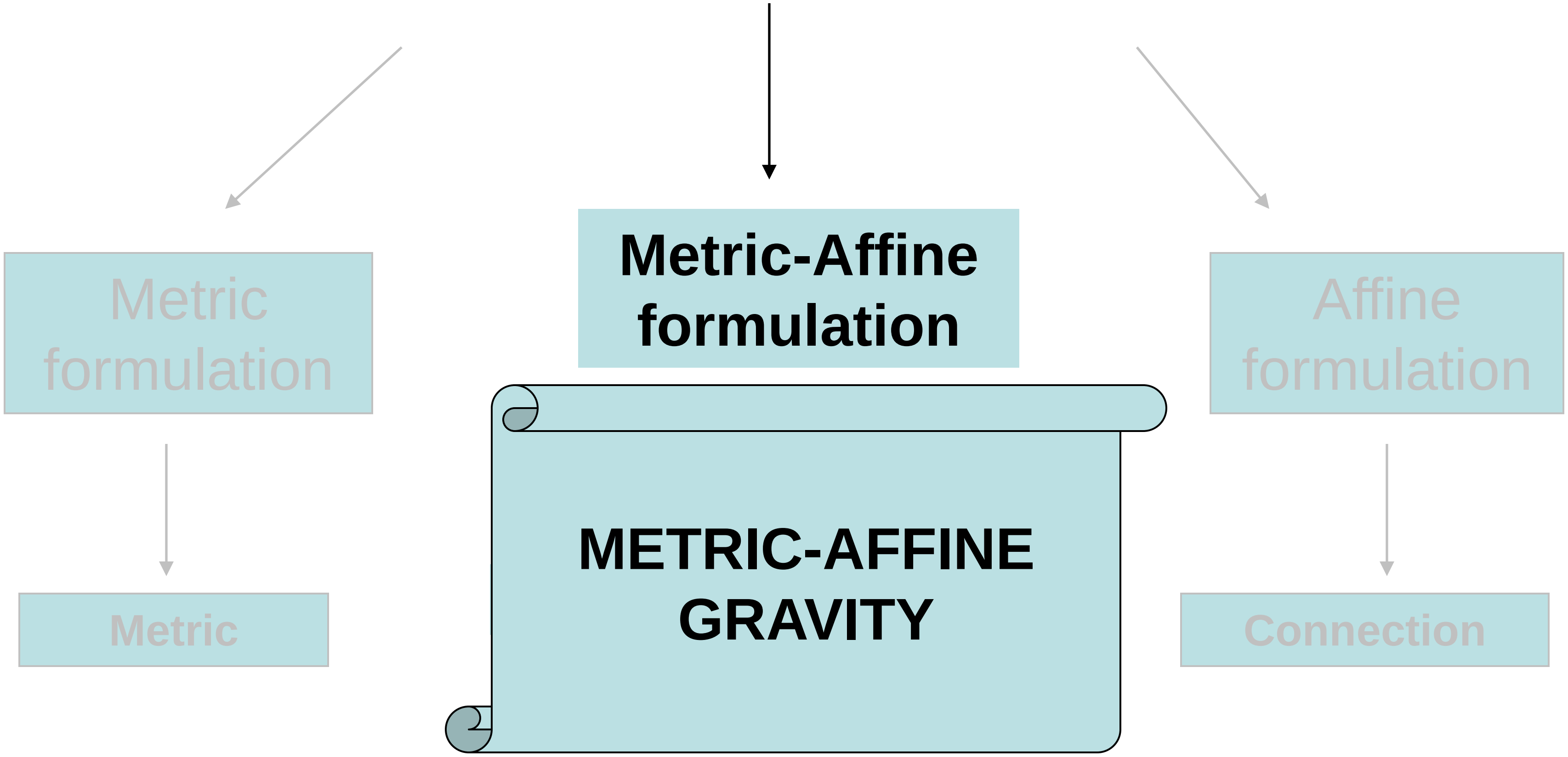
Metric-Affine Gravity

Formulations of gravity theories

- Metric formulation → metric
- Metric-Affine formulation → Metric and connection
- Affine formulation → Connection

Metric-Affine Gravity

formulations of gravity theories



- Metric formulation

$g_{\mu\nu}$ → **Metric compatibility**

$$\nabla_\alpha g_{\mu\nu} = 0$$

↓ **compatible**

$$\Gamma^{\lambda}_{\alpha\beta} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\rho\alpha} - \partial_\rho g_{\alpha\beta} \right)$$

Levi-Civita Connection

↓

$$\Gamma^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\nu\mu}$$

Torsion free

• Metric-Affine formulation

$g^{\alpha\beta}$ →

Non-Metricity

$$\mathbb{Q}_{\lambda}^{\alpha\beta}(g,\tilde{\Gamma}) = \nabla_{\lambda}^{\tilde{\Gamma}} g^{\alpha\beta}$$

↓ **Incompatible**

Connection is not Levi Civita connection

General Connection →

$$\tilde{\Gamma}^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\alpha\beta} + \Delta^{\lambda}_{\alpha\beta}$$

↓

Anti-symmetric Ricci tensor

$$\overline{\mathbb{R}}_{\alpha\beta}(\tilde{\Gamma}) \equiv \mathbb{R}^{\mu}_{\mu\alpha\beta}(\tilde{\Gamma}) = \partial_{\alpha}\tilde{\Gamma}^{\mu}_{\beta\mu} - \partial_{\beta}\tilde{\Gamma}^{\mu}_{\alpha\mu}$$

Torsion

$$\mathbb{S}^{\lambda}_{\alpha\beta}(\tilde{\Gamma}) = \tilde{\Gamma}^{\lambda}_{\alpha\beta} - \tilde{\Gamma}^{\lambda}_{\beta\alpha}$$

Non-Riemannian structures

Metric-Affine Formulation

Non-Riemannian structures :

- **Non-Metricity**
- **Torsion**
- **Anti-symmetric Ricci tensor**

General Connection

$$\mathbb{\Gamma}^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\alpha\beta} + \Delta^{\lambda}_{\alpha\beta}$$

Levi-Civita connection

Tensorial connection

↓

$$\mathbb{R}^{\mu}_{\alpha\nu\beta}(\mathbb{\Gamma}) = \partial_{\nu}\,\mathbb{\Gamma}^{\mu}_{\beta\alpha} - \partial_{\beta}\,\mathbb{\Gamma}^{\mu}_{\nu\alpha} + \mathbb{\Gamma}^{\mu}_{\nu\lambda}\mathbb{\Gamma}^{\lambda}_{\beta\alpha} - \mathbb{\Gamma}^{\mu}_{\beta\lambda}\mathbb{\Gamma}^{\lambda}_{\nu\alpha}$$

↓

$$\mathbb{R}_{\alpha\beta}(\mathbb{\Gamma}) = R_{\alpha\beta}(\Gamma) + \mathcal{R}_{\alpha\beta}(\Delta)$$

↓

$$\mathcal{R}_{\alpha\beta} = \nabla_{\mu}\Delta^{\mu}_{\beta\alpha} - \nabla_{\beta}\Delta^{\mu}_{\mu\alpha} + \Delta^{\mu}_{\mu\nu}\Delta^{\nu}_{\beta\alpha} - \Delta^{\mu}_{\beta\nu}\Delta^{\nu}_{\mu\alpha}$$

Tensor-Vector Theory

$$V_\alpha = \Delta^\mu_{\alpha\mu}$$

$$U_\alpha = \Delta^\mu_{\mu\alpha}$$

$$W^\alpha = g^{\mu\nu}\Delta^\alpha_{\mu\nu}$$

$$\begin{aligned}\Delta^\lambda_{\alpha\beta} = &\ \delta^\lambda_{\alpha\beta} + a_v V^\lambda g_{\alpha\beta} + b_v V_\alpha \delta^\lambda_\beta + c_v \delta^\lambda_\alpha V_\beta \\ &+ a_u U^\lambda g_{\alpha\beta} + b_u U_\alpha \delta^\lambda_\beta + c_u \delta^\lambda_\alpha U_\beta \\ &+ a_w W^\lambda g_{\alpha\beta} + b_w W_\alpha \delta^\lambda_\beta + c_w \delta^\lambda_\alpha W_\beta \\ &+ \frac{1}{M^2}\sum \left(\nu_{xy} V^\lambda + \upsilon_{xy} U^\lambda + \omega_{xy} W^\lambda\right) X_\alpha Y_\beta\end{aligned}$$

Higher spin field

Small contributions

$$a_v = c_v = a_u = b_u = b_w = c_w = -\frac{1}{18}$$

$$b_v = c_u = a_w = \frac{5}{18}$$

Using tensorial connection

$$\mathcal{R}(g,\Delta) = \nabla\cdot(\mathtt{W}-\mathtt{U}) + \frac{1}{18}\Big(\mathtt{V}\cdot\mathtt{V}+\mathtt{U}\cdot\mathtt{U}+\mathtt{W}\cdot\mathtt{W} - 4\mathtt{V}\cdot\mathtt{U} - 4\mathtt{V}\cdot\mathtt{W} + 14\mathtt{U}\cdot\mathtt{W}\Big)$$

$$\mathbb{Q}_{\lambda}^{\alpha\beta}(g,\between) = \Delta^{\alpha}_{\lambda\mu}g^{\mu\beta} + \Delta^{\beta}_{\lambda\mu}g^{\alpha\mu}$$

→

$$\mathbb{Q}_{\lambda}^{\alpha\beta} = \frac{1}{9}\Big(5\mathtt{V}_{\lambda}g^{\alpha\beta} - \mathtt{V}^{\alpha}\delta^{\beta}_{\lambda} - \delta^{\alpha}_{\lambda}\mathtt{V}^{\beta} - \mathtt{U}_{\lambda}g^{\alpha\beta} + 2\mathtt{U}^{\alpha}\delta^{\beta}_{\lambda} + 2\delta^{\alpha}_{\lambda}\mathtt{U}^{\beta} - \mathtt{W}_{\lambda}g^{\alpha\beta} + 2\mathtt{W}^{\alpha}\delta^{\beta}_{\lambda} + 2\delta^{\alpha}_{\lambda}\mathtt{W}^{\beta}\Big)$$

$$\mathbb{S}^{\lambda}_{\alpha\beta}(\between) = \Delta^{\lambda}_{\alpha\beta} - \Delta^{\lambda}_{\beta\alpha}$$

→

$$\mathbb{S}^{\lambda}_{\alpha\beta} = \frac{1}{3}\left(\mathtt{V}_{\alpha}\delta^{\lambda}_{\beta} - \delta^{\lambda}_{\alpha}\mathtt{V}_{\beta}\right) - \frac{1}{3}\left(\mathtt{U}_{\alpha}\delta^{\lambda}_{\beta} - \delta^{\lambda}_{\alpha}\mathtt{U}_{\beta}\right)$$

$$\overline{\mathbb{R}}_{\alpha\beta}(\between) \equiv \mathbb{R}^{\mu}{}_{\mu\alpha\beta}(\between) = \partial_{\alpha}\between^{\mu}_{\beta\mu} - \partial_{\beta}\between^{\mu}_{\alpha\mu}$$

→

$$\overline{\mathbb{R}}_{\alpha\beta}(\between) \equiv \mathtt{V}^{(-)}_{\alpha\beta} \equiv \partial_{\alpha}\mathtt{V}_{\beta} - \partial_{\beta}\mathtt{V}_{\alpha}$$

General action

$$\mathbb{R},\ \mathbb{S}\bullet\mathbb{S},\ \mathbb{Q}\bullet\mathbb{Q},\ \mathbb{Q}\bullet\mathbb{S},$$

→ Contraction of tensors in all possible ways

→ **Mass dimension-2 invariants**

$$\mathbb{R}^2,\ \mathbb{R}\bullet\mathbb{R},\ \overline{\mathbb{R}}\bullet\overline{\mathbb{R}},$$
$$\mathbb{R}\bullet\mathbb{S}\bullet\mathbb{S},\ \mathbb{R}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \mathbb{R}\bullet\mathbb{Q}\bullet\mathbb{S},$$
$$\overline{\mathbb{R}}\bullet\mathbb{S}\bullet\mathbb{S},\ \overline{\mathbb{R}}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \overline{\mathbb{R}}\bullet\mathbb{Q}\bullet\mathbb{S},$$
$$\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S},\ \mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q},$$
$$\mathbb{S}\bullet\mathbb{Q}\bullet\mathbb{Q}\bullet\mathbb{Q},\ \mathbb{S}\bullet\mathbb{S}\bullet\mathbb{Q}\bullet\mathbb{Q},$$
$$\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{S}\bullet\mathbb{Q},\ \mathcal{L}_{\tt matter}\left(g,\lozenge,\psi\right)$$

→ **Mass dimension-4 invariants**

$$\nabla^{\lozenge}\mathbb{Q}\bullet\nabla^{\lozenge}\mathbb{S} \qquad \nabla^{\lozenge}\mathbb{S}\bullet\nabla^{\lozenge}\mathbb{S} \qquad \nabla^{\lozenge}\mathbb{Q}\bullet\nabla^{\lozenge}\mathbb{Q}$$

$$I = \int d^4x\,\sqrt{-g}\,\left\{\mathfrak{L}\left(\mathbb{R},\overline{\mathbb{R}},\mathbb{S},\mathbb{Q}\right) + L_m\left(\psi,g\right) - V_0\right\}$$

$$\begin{aligned}\mathfrak{L} \;=\; & \frac{1}{2}M_{Pl}^2\left(\mathbb{R} + c_S\mathbb{S}\bullet\mathbb{S} + c_Q\mathbb{Q}\bullet\mathbb{Q} + c_{QS}\mathbb{Q}\bullet\mathbb{S}\right)\\ & + c'_S\nabla^{\lozenge}\mathbb{S}\bullet\nabla^{\lozenge}\mathbb{S} + c'_Q\nabla^{\lozenge}\mathbb{Q}\bullet\nabla^{\lozenge}\mathbb{Q} + c'_{QS}\nabla^{\lozenge}\mathbb{Q}\bullet\nabla^{\lozenge}\mathbb{S}\\ & + c_{R^2}\mathbb{R}^2 + c_{RR}\mathbb{R}\bullet\mathbb{R} + c_{\overline{RR}}\overline{\mathbb{R}}\bullet\overline{\mathbb{R}} + \mathcal{O}\left(\frac{1}{M_{Pl}^2}\right)\end{aligned}$$

Zero !!!

Ghosty higher derivative interactions

↓

The coefficients must be zero

$$\overline{\mathbb{R}} \bullet \overline{\mathbb{R}} = \mathtt{V}^{(-)\alpha\beta}\mathtt{V}^{(-)}_{\alpha\beta}$$

$$\mathbb{S} \bullet \mathbb{S} = 2\left(\mathtt{V}\cdot\mathtt{V} + \mathtt{U}\cdot\mathtt{U} - 2\mathtt{V}\cdot\mathtt{U}\right)$$

$$\mathbb{Q} \bullet \mathbb{Q} = \frac{2}{9}\Big(22\mathtt{V}\cdot\mathtt{V} + 7\mathtt{U}\cdot\mathtt{U} + 7\mathtt{W}\cdot\mathtt{W} + 20\mathtt{V}\cdot\mathtt{U} + 20\mathtt{V}\cdot\mathtt{W} + 14\mathtt{U}\cdot\mathtt{W}\Big)$$

$$\mathbb{Q} \bullet \mathbb{S} = \frac{4}{3}\Big(2\mathtt{V}\cdot\mathtt{V} + \mathtt{U}\cdot\mathtt{U} - 3\mathtt{V}\cdot\mathtt{U} - \mathtt{V}\cdot\mathtt{W} + \mathtt{U}\cdot\mathtt{W}\Big)$$

$$\mathbb{D}_{\alpha\beta} = \nabla^{\between}_{\lambda}\mathbb{S}^{\lambda}_{\alpha\beta} \supset -\frac{1}{3}\mathtt{V}^{(-)}_{\alpha\beta} + \frac{1}{3}\mathtt{U}^{(-)}_{\alpha\beta}$$

$$\mathbb{D} \bullet \mathbb{D} \supset \frac{1}{9}\Big(\mathtt{V}^{(-)\alpha\beta}\mathtt{V}^{(-)}_{\alpha\beta} + \mathtt{U}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta} - 2\mathtt{V}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta}\Big)$$

$$\mathtt{A}^{(-)}_{\alpha\beta} = \partial_\alpha \mathtt{A}_\beta - \partial_\beta \mathtt{A}_\alpha$$

$$\mathbb{N}^{\alpha\beta} = g^{\rho\lambda}\nabla^{\between}_{\rho}\mathbb{Q}^{\alpha\beta}_{\lambda} \supset \frac{1}{9}\Big(5\nabla\cdot\mathtt{V}g^{\alpha\beta} - \mathtt{V}^{(+)\alpha\beta} - \nabla\cdot\mathtt{U}g^{\alpha\beta} + 2\mathtt{U}^{(+)\alpha\beta} - \nabla\cdot\mathtt{W}g^{\alpha\beta} + 2\mathtt{W}^{(+)\alpha\beta}\Big)$$

$$\mathbb{N} \bullet \mathbb{N} \supset \frac{1}{162}\sum_{i,j=1}^{3}\mathtt{A}^{(+)}_{i\alpha\beta}\mathtt{K}^{\alpha\beta\mu\nu}_{ij}\mathtt{A}^{(+)}_{j\mu\nu}$$

$$\mathtt{A}_i \in (\mathtt{V}, \mathtt{U}, \mathtt{W})$$

$$\mathtt{A}^{(+)}_{\alpha\beta} \equiv \nabla_\alpha \mathtt{A}_\beta + \nabla_\beta \mathtt{A}_\alpha$$

$$\mathtt{K}^{\alpha\beta\mu\nu} = \begin{pmatrix} 202g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu} & g^{\alpha\beta}g^{\mu\nu} - 2g^{\alpha\mu}g^{\beta\nu} - 2g^{\alpha\nu}g^{\beta\mu} & g^{\alpha\beta}g^{\mu\nu} - 2g^{\alpha\mu}g^{\beta\nu} - 2g^{\alpha\nu}g^{\beta\mu} \\ g^{\alpha\beta}g^{\mu\nu} - 2g^{\alpha\mu}g^{\beta\nu} - 2g^{\alpha\nu}g^{\beta\mu} & -2g^{\alpha\beta}g^{\mu\nu} + 4g^{\alpha\mu}g^{\beta\nu} + 4g^{\alpha\nu}g^{\beta\mu} & -2g^{\alpha\beta}g^{\mu\nu} + 4g^{\alpha\mu}g^{\beta\nu} + 4g^{\alpha\nu}g^{\beta\mu} \\ g^{\alpha\beta}g^{\mu\nu} - 2g^{\alpha\mu}g^{\beta\nu} - 2g^{\alpha\nu}g^{\beta\mu} & -2g^{\alpha\beta}g^{\mu\nu} + 4g^{\alpha\mu}g^{\beta\nu} + 4g^{\alpha\nu}g^{\beta\mu} & -2g^{\alpha\beta}g^{\mu\nu} + 4g^{\alpha\mu}g^{\beta\nu} + 4g^{\alpha\nu}g^{\beta\mu} \end{pmatrix}$$

$$
\begin{aligned}
I &= \int d^4x\, \sqrt{-g} \left\{ \frac{1}{2} M_{Pl}^2 R + L_m(\psi, g) - V_0 \right\} \\
&+ \int d^4x\, \sqrt{-g} \Big\{ c_{VV} \mathtt{V}^{(-)\alpha\beta} \mathtt{V}^{(-)}_{\alpha\beta} + c_{UU} \mathtt{U}^{(-)\alpha\beta} \mathtt{U}^{(-)}_{\alpha\beta} + c_{VU} \mathtt{V}^{(-)\alpha\beta} \mathtt{U}^{(-)}_{\alpha\beta} \\
&+ \mathtt{V}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{VV} \mathtt{V}^{(+)}_{\mu\nu} + \mathtt{U}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{UU} \mathtt{U}^{(+)}_{\mu\nu} + \mathtt{W}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{WW} \mathtt{W}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{VU} \mathtt{U}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{VW} \mathtt{W}^{(+)}_{\mu\nu} + \mathtt{U}^{(+)}_{\alpha\beta} \mathtt{k}^{\alpha\beta\mu\nu}_{UW} \mathtt{W}^{(+)}_{\mu\nu} \\
&+ M_{Pl}^2 \left(\frac{1}{2} a_{VV} \mathtt{V}^{\alpha} \mathtt{V}_{\alpha} + \frac{1}{2} a_{UU} \mathtt{U}^{\alpha} \mathtt{U}_{\alpha} + \frac{1}{2} a_{WW} \mathtt{W}^{\alpha} \mathtt{W}_{\alpha} + a_{VU} \mathtt{V}^{\alpha} \mathtt{U}_{\alpha} + a_{VW} \mathtt{V}^{\alpha} \mathtt{W}_{\alpha} + a_{UW} \mathtt{U}^{\alpha} \mathtt{W}_{\alpha} \right) \Big\}
\end{aligned}
$$

Tensor-Vector Theory = GR + a theory of three vectors

$$
\begin{aligned}
a_{VV} &= \frac{1}{18} + 2c_S + \frac{44}{9} c_Q + \frac{8}{3} c_{QS} \\
a_{UU} &= c_{WW} + 2c_S + \frac{4}{3} c_{QS}\,, \\
a_{WW} &= \frac{1}{18} + \frac{14}{9} c_Q\,, \\
a_{VU} &= -\frac{1}{9} - 2c_S + \frac{20}{9} c_Q - 2c_{QS} \\
a_{VW} &= -\frac{1}{9} + \frac{20}{9} c_Q - \frac{2}{3} c_{QS} \\
a_{UW} &= \frac{7}{18} + \frac{14}{9} c_Q + \frac{2}{3} c_{QS}
\end{aligned}
$$

- **Two different type of kinetic terms** :

$$A^{(-)}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$ → Gauge-like fields → **U and V fields**

$$A^{(+)}_{\alpha\beta} \equiv \nabla_\alpha A_\beta + \nabla_\beta A_\alpha$$ → Non-gauge-like fields → **U,V and W fields**

Vector fields are not associated with a gauge theory

↓

They are not vectors originating from need to realize local U(1) invariance

Limiting cases

1. **Symmetric connection**

$$\between^{\lambda}_{\alpha\beta} = \between^{\lambda}_{\beta\alpha} \longrightarrow \quad \mathbb{S}^{\lambda}_{\alpha\beta} = 0, \quad V_{\alpha} = U_{\alpha}$$

→ **Theory of two vector fields**

2. **Anti-symmetric tensorial connection**

$$\Delta^{\lambda}_{\alpha\beta} = -\Delta^{\lambda}_{\alpha\beta} \longrightarrow \quad W_{\alpha} = 0, \quad V_{\alpha} = -U_{\alpha}$$

→ **Theory of a single vector field**

TeVeS from Metric-Affine Gravity

– **General Relativity fails to account for dynamics at galactic scales without postulating a large amount of CDM**

Newton's second Law → $$\mu\left(\frac{|\vec{a}|}{a_0}\right)\vec{a} = -\vec{\nabla}\Phi_N$$ **MOND**

$$\mu(x) \rightsquigarrow 1(x)$$

$$x \gg 1 (x \ll 1)$$

$$a_0 \simeq 10^{-10}\ m/s^2$$

Gravitational potential

Relativistic limit (left) ↓ **General Relativity**

Relativistic limit (right) ↓ **TeVeS**

MOND

↓ **Relativistic Generalization**

TeVeS

bi-metrical gravitational theory

Vector and scalar fields in addition to metric tensor

$$\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu} - 2\sinh(2\phi) A_\mu A_\nu$$

Scalar Field (ϕ) **Vector Fields** (A_μ, A_ν)

TeVeS Action Ingredients

$$S_{\tilde{g}} = \frac{1}{16\pi G}\int d^4x(-g)^{\frac{1}{2}}\left[R + K^{abmn}\nabla_a A_m \nabla_b A_n\right]$$

$$\begin{aligned} K^{abmn} &= d_1 g^{ab} g^{mn} + d_2 g^{am} g^{bn} + d_3 g^{an} g^{bm} \\ &+ d_4 A^a A^b g^{mn} + d_5 g^{an} A^b A^m + d_6 g^{ab} A^m A^n \\ &+ d_7 g^{am} A^b A^n + d_8 A^a A^b A^m A^n \end{aligned}$$

$$S_s = -\frac{1}{16\pi G}\int d^4x(-\tilde{g})^{\frac{1}{2}}\left[\mu(\tilde{g}^{ab} - A^a A^b)\tilde{\nabla}_a\phi\tilde{\nabla}_b\phi + V(\mu)\right]$$

$$S_v = -\frac{1}{32\pi G}\int d^4x(-\tilde{g})^{\frac{1}{2}}\left[K F^{ab} F_{ab} - 2\lambda(\tilde{g}^{ab} A_a A_b + 1)\right]$$

$$F_{ab} = 2\tilde{\nabla}_{[a} A_{b]}$$

$$
\begin{aligned}
I = &\int d^4x\,\sqrt{-g}\left\{\frac{1}{2}M_{Pl}^2 R + L_m(\psi,g) - V_0\right\}\\
&+ \int d^4x\,\sqrt{-g}\Big\{c_{VV}\mathtt{V}^{(-)\alpha\beta}\mathtt{V}^{(-)}_{\alpha\beta} + c_{UU}\mathtt{U}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta} + c_{VU}\mathtt{V}^{(-)\alpha\beta}\mathtt{U}^{(-)}_{\alpha\beta}\\
&+ \mathtt{V}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{VV}\mathtt{V}^{(+)}_{\mu\nu} + \mathtt{U}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{UU}\mathtt{U}^{(+)}_{\mu\nu} + \mathtt{W}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{WW}\mathtt{W}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{VU}\mathtt{U}^{(+)}_{\mu\nu} + \mathtt{V}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{VW}\mathtt{W}^{(+)}_{\mu\nu} + \mathtt{U}^{(+)}_{\alpha\beta}\mathtt{k}^{\alpha\beta\mu\nu}_{UW}\mathtt{W}^{(+)}_{\mu\nu}\\
&+ M_{Pl}^2\left(\frac{1}{2}a_{VV}\mathtt{V}^{\alpha}\mathtt{V}_{\alpha} + \frac{1}{2}a_{UU}\mathtt{U}^{\alpha}\mathtt{U}_{\alpha} + \frac{1}{2}a_{WW}\mathtt{W}^{\alpha}\mathtt{W}_{\alpha} + a_{VU}\mathtt{V}^{\alpha}\mathtt{U}_{\alpha} + a_{VW}\mathtt{V}^{\alpha}\mathtt{W}_{\alpha} + a_{UW}\mathtt{U}^{\alpha}\mathtt{W}_{\alpha}\right)\Big\}
\end{aligned}
$$

Constraint : Anti-symmetric tensorial connection

$$\Delta^{\lambda}_{\ \alpha\beta} = -\Delta^{\lambda}_{\ \alpha\beta}$$

$$\mathtt{W}_{\alpha} = 0$$

$$\mathtt{V}_{\alpha} = -\mathtt{U}_{\alpha}$$

$$
\begin{aligned}
I \;=\; & \int d^4x\, \sqrt{-g}\Bigg\{ \frac{1}{2} M_{Pl}^2 R + L_m(\psi, g) - V_0 \\
+ \; & \overline{c}_{VV}\, \mathrm{V}^{(-)\alpha\beta} \mathrm{V}^{(-)}_{\alpha\beta} + \mathrm{V}^{(+)}_{\alpha\beta} \overline{\mathrm{k}}^{\alpha\beta\mu\nu}_{VV} \mathrm{V}^{(+)}_{\mu\nu} \\
+ \; & \frac{1}{2} M_{Pl}^2 \overline{a}_{VV}\, \mathrm{V}^{\alpha} \mathrm{V}_{\alpha} \Bigg\}
\end{aligned}
$$

$$
\overline{a}_{VV} = \frac{1}{3} + 8c_S + 2c_Q + 8c_{QS}
$$

$$\mathrm{V}_\alpha = a_1 V_\alpha + \frac{a_0}{M_{Pl}}\partial_\alpha\phi$$

$$\Downarrow$$

$$\begin{aligned} I &= \int d^4x\,\sqrt{-g}\Big\{\frac{1}{2}M_{Pl}^2 R + L_m(\psi,g) - V_0 \\ &+ a_1^2\overline{\mathrm{c}}_{VV}V^{(-)\alpha\beta}V^{(-)}_{\alpha\beta} + a_1^2 V^{(+)}_{\alpha\beta}\overline{\mathrm{k}}^{\alpha\beta\mu\nu}_{VV}V^{(+)}_{\mu\nu} \\ &+ \frac{1}{2}M_{Pl}^2 a_1^2\overline{\mathrm{a}}_{VV}V^\alpha V_\alpha + M_{Pl}a_1 a_0\overline{\mathrm{a}}_{VV}V^\alpha\partial_\alpha\phi \\ &+ a_0^2\overline{\mathrm{a}}_{VV}\partial^\alpha\phi\partial_\alpha\phi + \mathcal{O}\left(\frac{1}{M_{Pl}}\right)\Big\} \end{aligned}$$

$$a_0 = \bar{a}_0\mu \qquad\qquad V_0 = V(\mu) + \Delta V$$

$$\downarrow$$

TeVeS like Gravity

$$S_{\tilde{g}} = \frac{1}{16\pi G}\int d^4x(-g)^{\frac{1}{2}}\left[R + K^{abmn}\nabla_a A_m \nabla_b A_n\right]$$

$$I = \int d^4x\,\sqrt{-g}\Big\{\frac{1}{2}M_{Pl}^2 R + a_1^2 V_{\alpha\beta}^{(+)}\overline{\mathrm{k}}_{VV}^{\alpha\beta\mu\nu}V_{\mu\nu}^{(+)} + L_m(\psi, g) - \Delta V$$
$$+ a_1^2\overline{c}_{VV}V^{(-)\alpha\beta}V_{\alpha\beta}^{(-)} + \frac{1}{2}M_{Pl}^2 a_1^2\overline{a}_{VV}V^\alpha V_\alpha$$
$$+ M_{Pl}a_1\bar{a}_0\mu\overline{a}_{VV}V^\alpha\partial_\alpha\phi + \bar{a}_0^2\mu^2\overline{a}_{VV}\partial^\alpha\phi\partial_\alpha\phi - V(\mu)\Big\}$$

$$S_s = -\frac{1}{16\pi G}\int d^4x(-\tilde{g})^{\frac{1}{2}}\left[\mu(\tilde{g}^{ab} - A^a A^b)\tilde{\nabla}_a\phi\tilde{\nabla}_b\phi + V(\mu)\right]$$

$$S_v = -\frac{1}{32\pi G}\int d^4x(-\tilde{g})^{\frac{1}{2}}\left[KF^{ab}F_{ab} - 2\lambda(\tilde{g}^{ab}A_a A_b + 1)\right]$$

Conclusion

- Metric-Affine Gravity generalize the GR by accommodating an affine connection that extends the Levi-Civita connection
- Tensorial part of connection, under general condition can be decomposed into three independent vector fields
- Tensor,vector come into action in a natural way
- The resulting tensor-vector theory is rather general
- By imposing judicious constraints, theory can be reduced to more familiar ones like TeVeS gravity

Conclusion

Metric-affine gravity is rich enough to supply various vector and scalar fields needed in cosmological phenomena !!!

Thanks to my collaborators :

• Aslı ALTAŞ

• Durmuş Ali DEMİR

arXiv:1110.5168v1 [gr-qc]

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