Termal KRD Toplam Kuralları ile Ağır - Ağır Mezonların İncelenmesi

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Introduction

Investigations of mesons play the essential role in understanding the vacuum properties of the nonperturbative QCD [1]-[3]. In particular, analysis of the variation of the mesons parameters in hadronic medium with respect to the temperature can give information about the QCD vacuum and transition to the quark gluon plasma (QGP) phase. Determination of the hadronic properties of mesons in hot and dense QCD medium has become one of the most important research subject in the last twenty years both theoretically and experimentally. Properties of the light, heavy-light and heavy mesons in vacuum have been investigated widely in the literature using the nonperturbative approaches like QCD sum rules, nonrelativistic potential models, lattice theory, heavy quark effective theory and chiral perturbation theory.

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Introduction

QCD sum rules which is based on the operator product expansion (OPE), QCD Lagrangian and quark-hadron duality, is one of the most informative, applicable and predictive models in hadron physics [4]. The thermal version of this model proposed by Bochkarev and Shaposhnikov [5]. In expansion of these models to finite temperature we are face to face with some difficulties [6-8]. One of the new feature is the interaction of the current with the particles in the medium which requires the modification of the dispersion representation. The other new feature of the thermal QCD is breakdown of the Lorentz invariance via the choice of reference frame. Due to residual O(3) symmetry at finite temperature, more operators with the same dimensions appear in the OPE comparing to the QCD sum rules in vacuum. Thermal version of QCD sum rules has been successfully used to study the thermal properties of light, heavy-light and heavy-heavy mesons as well-established method.

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We investigate the masses and leptonic decay constants of the heavy-heavy scalar, pseudoscalar and vector mesons in the framework of finite temperature QCD sum rules. The annihilation and scattering parts of spectral density are calculated in the lowest order of perturbation theory. Taking into account the additional operators arising at finite temperature, the nonperturbative corrections are also evaluated. The investigations show that the thermal contributions are significantly important and the obtained results at zero temperature are in good consistency with the existing experimental data as well as predictions of the other nonperturbative models.

To obtain the thermal QCD sum rules for physical quantities, we need to calculate the convenient thermal correlation function in two different ways: in terms of QCD degrees of freedom and in terms of hadronic parameters. In QCD side, the correlation function is evaluated via OPE which helps us expand the time ordering product of currents in terms of operators with different dimensions. We begin by considering the following two point thermal correlation function:

$$\Pi(q,T) = i\int d^4x e^{iq.x}\langle \mathcal{T}\left(J(x)J^{+}(0)\right)\rangle . \tag{1}$$

In correlation function $\mathcal{T}$ is the time ordering product and where $J(x) = \bar{q}_1(x)\Gamma q_2(x)$ is the interpolating current that carries the quantum numbers of the state concerned. Here $\Gamma = I$ or $i\gamma_5$ for scalar and pseudoscalar particles, respectively and $\Gamma = \gamma_\mu$ as for vector particles. The thermal average of any operator, A can be expressed as:

$$\langle A\rangle_\beta = Z^{-1}(\beta)Tr\rho(\beta)A = \frac{Tr(e^{-\beta H}A)}{Tr(e^{-\beta H})}, \tag{2}$$

where $\rho(\beta)$ is density matrix for the system, H is the QCD Hamiltonian, and $\beta = 1/T$ is the inverse of the temperature T and traces are carried out over any complete set of states.

The thermal average of the correlation function of any two operators,A and B, with different coordinates can similarly be written as

$$\langle AB\rangle_{\beta} = Z^{-1}(\beta)\, Tr\rho(\beta)AB, \tag{3}$$

For any arbitrary Schrödinger operator A, we have the Heisenberg operator $A_H(t)$ defined as

$$A_H(t) = e^{iHt}Ae^{-iHt}, \tag{4}$$

It is clear now that, for a general thermal correlation function of two Heisenberg operators $A_H(t)$ and $B_H(t')$, we can write

$$\begin{aligned}
\langle A_H(t)B_H(t')\rangle_{\beta} &= Z^{-1}(\beta)\, Tr\rho(\beta)A_H(t)B_H(t') \\
&= Z^{-1}(\beta)\, Tr e^{-\beta H}A_H(t)e^{\beta H}e^{-\beta H}B_H(t') \\
&= Z^{-1}(\beta)\, Tr A_H(t+i\beta)e^{-\beta H}B_H(t') \\
&= Z^{-1}(\beta)\, Tr e^{-\beta H}B_H(t')A_H(t+i\beta) \\
&= \langle B_H(t')A_H(t+i\beta)\rangle_{\beta}, \qquad (5)
\end{aligned}$$

Kubo-Martin-Schwinger relation

The Green function for scalar fields is defined as the following form

$$(\partial_{\mu\partial^{\mu}} + m^2)G(\vec{x} - \vec{y}, t - t') = -\delta^3(x - y)\delta(t - t'). \tag{6}$$

The solution of this, subject to the periodicity condition (see Eq. (5)) can be easily determined to be

$$\begin{aligned} G(t - t', \omega) &= \frac{n_B(\omega)}{2i\omega}[\theta(t - t')(e^{\beta\omega - i\omega(t-t')} + e^{i\omega(t-t')}) \\ &+ \theta(t - t')(e^{-i\omega(t-t')} + e^{\beta\omega + i\omega(t-t')})]. \end{aligned} \tag{7}$$

where $\omega_k = (k^{\vec{2}} + m^2)^{1/2}$ The finite temperature Greens functions for scaler field and free Dirac field as the following forms, respectively

$$G(p) = \Big(\frac{1}{p^2 - m^2 + i\varepsilon} - 2i\pi n_B(|p^0|)\delta(p^2 - m^2)\Big) \tag{8}$$

and

$$S(k) = (\gamma^{\mu}k_{\mu} + m)\Big(\frac{1}{k^2 - m^2 + i\varepsilon} + 2\pi i n(|k_0|)\delta(k^2 - m^2)\Big), \tag{9}$$

The fundamental assumption of Wilson expansion is that the product of operators at different points can be expanded as the sum of local operators with momentum dependent coefficients in the form:

$$T(q) = \sum C_n(q^2)\langle O_n\rangle, \tag{10}$$

where $C_n(q^2)$ are called Wilson coefficients and O_n are a set of local operators. In this expansion, the operators are ordered according to their dimension d. The lowest dimension operator with $d = 0$ is the unit operator associated with the perturbative contribution. In the vacuum sum rules low dimension operators composed of quark and gluon fields are quark condensate $\langle\overline{\psi}\psi\rangle$ and gluon condensate $\langle G^a_{\mu\nu} G^{a\mu\nu}\rangle$. At finite temperature Lorentz invariance is broken by the choice of a preferred frame of reference and new operators appear in the Wilson expansion. To restore Lorentz invariance in thermal field theory, four-vector velocity of the medium u^μ is introduced. Using four-vector velocity and quark/gluon fields, we can construct a new set of low dimension operators $\langle u\Theta^f u\rangle$ and $\langle u\Theta^g u\rangle$ with dimension $d = 4$. So, we can write thermal correlation function in terms of operators up to dimension four:

$$T(q) = C_1 I + C_2\langle\overline{\psi}\psi\rangle + C_3\langle G^a_{\mu\nu} G^{a\mu\nu}\rangle + C_4\langle u\Theta^f u\rangle + C_5\langle u\Theta^g u\rangle. \tag{11}$$

In QCD side, the correlation function is calculated in deep Euclidean region, $q^2 \ll -\Lambda^2_{QCD}$ via OPE where the short or perturbative and long distance or non-perturbative effects are separated, i.e.,

$$\Pi^{QCD}(q,T) = \Pi^{pert}(q,T) + \Pi^{nonpert}(q,T). \tag{12}$$

The time ordering product in Eq. (1) can be expressed as

$$\langle T(J(x)J^{+}(x'))\rangle = \theta(x_0 - x_0')\langle J(x)J^{+}(x')\rangle + \theta(x_0' - x_0)\langle J^{+}(x')J(x)\rangle, \tag{13}$$

where $\theta(x)$ is step function.
Using Kubo-Martin-Schwinger relation, $\langle J(x_0)J^{+}(x_0')\rangle = \langle J^{+}(x_0')J(x_0 + i\beta)\rangle$ for thermal expectation and making Fourier and some other transformations, we get the following expression for the thermal correlation function in momentum space:

$$\Pi(|\mathbf{q}|, q_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dq_0' M(|\mathbf{q}|, q_0')\Big(\frac{1}{q_0 - q_0' + i\varepsilon} - \frac{\exp(-\beta q_0)}{q_0 - q_0' - i\varepsilon}\Big), \tag{14}$$

where

$$M(|\mathbf{q}|, q_0) = \int d^4x e^{iq\cdot x}\langle J(x)J^{+}(0)\rangle. \tag{15}$$

In the above transformations, the following standard integral representation for the θ-step function is used:

$$\theta(x_0 - x_0') = \frac{1}{2i\pi} \int_{-\infty}^{\infty} dk_0 \frac{\exp[ik_0(x_0 - x_0')]}{k_0 - i\varepsilon}. \quad (16)$$

The imaginary part of the correlation function can be simply evaluated using the formula $\frac{i}{x+i\varepsilon} = \pi\delta(x) + iP(\frac{1}{x})$, which leads to:

$$\Pi(q, T) = \int_0^{\infty} ds \frac{\rho(s)}{s + Q_0^2}, \quad (17)$$

where, $\rho(s, T)$ is called the spectral density at finite temperature. The thermal spectral density at fixed $\mid \mathbf{q} \mid$ can be expressed as:

$$\rho(q, T) = \frac{1}{\pi} \, Im\Pi^{pert}(q, T) \, \tanh\left(\frac{\beta q_0}{2}\right). \quad (18)$$

and $Q_0^2 = -q_0^2$.

The Lorentz invariance breaks down via the choice of reference frame at which the matter is at rest. However, using the four velocity vector u_μ of the matter, we can define Lorentz invariant quantities such as $\omega = u \cdot q$ and $\overline{q}^2 = \omega^2 - q^2$. By the help of these quantities, the thermal correlation function for heavy vector meson can be expressed in terms of two independent tensors $P_{\mu\nu}$ and $Q_{\mu\nu}$ at finite temperature [9], i.e.,

$$\Pi_{\mu\nu}\Big(q, T\Big) = Q_{\mu\nu}\Pi_l(q^2,\omega) + P_{\mu\nu}\Pi_t(q^2,\omega), \tag{19}$$

where

$$\begin{aligned} P_{\mu\nu} &= -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{q^2}{\overline{q}^2}\widetilde{u}_\mu\widetilde{u}_\nu, \\ Q_{\mu\nu} &= \frac{q^4}{\overline{q}^2}\widetilde{u}_\mu\widetilde{u}_\nu, \end{aligned} \tag{20}$$

and $\widetilde{u}_\mu = u_\mu - \omega\, q_\mu / q^2$.

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Here the functions Π_l and Π_t are the following Lorentz invariant functions:

$$\Pi_l\left(q^2,\omega\right) = \frac{1}{\overline{q}^2} u^\mu \Pi_{\mu\nu} u^\nu, \tag{21}$$

$$\Pi_t\left(q^2,\omega\right) = -\frac{1}{2}\left(g^{\mu\nu}\Pi_{\mu\nu} + \frac{q^2}{\overline{q}^2} u^\mu \Pi_{\mu\nu} u^\nu\right). \tag{22}$$

It can be shown that in the limit $|\mathbf{q}| \to 0$, the Π_t function can be expressed as $\Pi_t = -\frac{1}{3} g^{\mu\nu}\Pi_{\mu\nu}$ and one can easily find the $\Pi_t\left(q_0, |\mathbf{q}| = 0\right) = q_0^2\, \Pi_l\left(q_0, |\mathbf{q}| = 0\right)$ relation between two Π_l and Π_t functions. In real time thermal field theory, the function $\Pi_l(q^2,\omega)$ or $\Pi_t(q^2,\omega)$ can be written in 2×2 matrix form and elements of this matrix depend on only one analytic function [10]. Therefore, calculation of the 11-component of this matrix is sufficient to determine completely the dynamics of the corresponding two-point function.

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The thermal correlation function of Eq. (1) can be written in momentum space as:

$$\Pi_{\mu\nu}\Big(q,T\Big) = i\int \frac{d^4k}{(2\pi)^4} Tr\Big[\gamma_\mu S(k)\gamma_\nu S(k-q)\Big], \tag{23}$$

where, $S(k)$ is thermal quark propagator :

$$S(k) = (\gamma^\mu k_\mu + m)\Big(\frac{1}{k^2 - m^2 + i\varepsilon} + 2\pi i n(|k_0|)\delta(k^2 - m^2)\Big), \tag{24}$$

where $n(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi distribution function [11]. Now, we insert the propagator of Eq.(9) in Eq. (8) and consider $\Pi_1(q,T) = g^{\mu\nu}\Pi_{\mu\nu}(q,T)$ function.

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Carrying out the integral over k_0, we obtain the imaginary part of the $\Pi_1(q,T)$ in the following form:

$$Im\Pi_1(q,T) = L(q_0) + L(-q_0), \tag{25}$$

where

$$\begin{aligned} L(q_0) &= N_c \int \frac{d\mathbf{k}}{4\pi^2} \frac{\omega_1^2 - \mathbf{k}^2 + \mathbf{k}\cdot\mathbf{q} - \omega_1 q_0 - 2m^2}{\omega_1\omega_2} \\ &\times \Big([(1-n_1)(1-n_2) + n_1 n_2]\delta(q_0 - \omega_1 - \omega_2) - [(1-n_1)n_2 + (1-n_2)n_1] \\ &\times \delta(q_0 - \omega_1 + \omega_2)\Big), \end{aligned} \tag{26}$$

and $n_1 = n(\omega_1)$, $n_2 = n(\omega_2)$, $\omega_1 = \sqrt{\mathbf{k}^2 + m^2}$ and $\omega_2 = \sqrt{(\mathbf{k}-\mathbf{q})^2 + m^2}$. The terms without the Fermi distribution functions show the vacuum contributions but those including the Fermi distribution functions depict medium contributions. The delta-functions in the different terms of above Eq. control the regions of non-vanishing imaginary parts of $\Pi_1(q,T)$, which define the position of branch cuts [5].

As seen the term including $\delta(q_0 - \omega_1 - \omega_2)$ gives contribution when $q_0 = \omega_1 + \omega_2$. Using Cauchy-Schwarz inequality, $(\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2) \geq (\sum_{i=1}^{n} a_i b_i)^2$ we see that,

$$\omega_1\omega_2 = \sqrt{\mathbf{k}^2 + m^2}\sqrt{(\mathbf{k}\text{-}\mathbf{q})^2 + m^2} \geq |\mathbf{k}||\mathbf{k}\text{-}\mathbf{q}| + m^2, \tag{27}$$

Therefore, we obtain the first branch cut, $q^2 \geq 4m^2$, which coincides with zero temperature cut describing the standard threshold for particle decays. This term survives at zero temperature and it is called the annihilation term.

On the other hand, the term including $\delta(q_0 - \omega_1 + \omega_2)$ gives contribution when $q_0 = \omega_1 - \omega_2$. Similarly to the above expression, we obtain,

$$q_0^2 = 2m^2 + \mathbf{k}^2 + (\mathbf{k}\text{-}\mathbf{q})^2 - 2\omega_1\omega_2 \leq \mathbf{q}^2, \tag{28}$$

and therefore an additional branch cut arises at finite temperature, $q^2 \leq 0$, which corresponds to particle absorption from the medium. It is called scattering term and vanishes at $T = 0$.

After straightforward calculations, the annihilation and scattering parts of $\rho_1\left(q_0^2, T\right) = \frac{1}{\pi} Im\Pi_1\left(q_0^2, T\right) \tanh\frac{\beta q_0}{2}$ at nonzero momentum can be written as:

$$\rho_{1,a} = \frac{-3q^2}{8\pi^2}(3 - v^2)\left[v - \int_{-v}^{v} dx\, n_+(x)\right] \qquad \text{for} \qquad 4m^2 + \mathbf{q}^2 \leq q_0^2 \leq \infty, \qquad (29)$$

$$\rho_{1,s} = \frac{3q^2}{16\pi^2}(3 - v^2)\int_{v}^{\infty} dx\left[n_-(x) - n_+(x)\right] \qquad \text{for} \qquad q_0^2 \leq \mathbf{q}^2, \qquad (30)$$

where $v(q_0^2) = \sqrt{1 - 4m^2/q_0^2}$, $n_+(x) = n\left[\frac{1}{2}(q_0 + |\mathbf{q}|x)\right]$ and $n_-(x) = n\left[\frac{1}{2}(|\mathbf{q}|x - q_0)\right]$.

From the similar manner, we can calculate also the function $\Pi_2(q,T) = u^\mu \Pi_{\mu\nu}(q,T) u^\nu$. After some simplifications, we obtain the imaginary part of the $\Pi_2(q,T)$ in the form

$$Im\Pi_2(q,T) = -4iN_c \int \frac{d^4k}{(2\pi)^4}(k^2 - q\cdot k - m^2 + 2q_0k_0 - 2k_0^2)D(k)D(k-q), \quad (31)$$

where $D(k) = 1/(k^2 - m^2 + i\varepsilon) + 2\pi i n(|k_0|)\delta(k^2 - m^2)$. Carrying out the integral over k_0 and angles we obtain annihilation and scattering parts of $Im\Pi_2(q,T)$ as follows:

$$Im\Pi_{2,a} = N_c \int_{\omega_-}^{\omega_+} \frac{d\omega_1}{8\pi|\mathbf{q}|}(4q_0\omega_1 - q^2 - 4\omega_1^2)F(\omega_1), \quad (32)$$

$$Im\Pi_{2,s} = N_c \int_{\omega_+}^{\infty} \frac{d\omega_1}{8\pi|\mathbf{q}|}(4q_0\omega_1 - q^2 - 4\omega_1^2)G(\omega_1). \quad (33)$$

Here $F(\omega_1) = 1 - n(\omega_1) - n(q_0 - \omega_1) + 2n(\omega_1)n(q_0 - \omega_1)$, $G(\omega_1) = 2n(\omega_1)n(q_0 - \omega_1) - n(\omega_1) - n(q_0 - \omega_1)$ and $\omega_\pm = \frac{1}{2}(q_0 \pm |\mathbf{q}|v)$.

In rest frame, taking into account as follows,

$$Im\Pi_{l,a} = \frac{1}{|\vec{q}|^2} Im\Pi_{2,a}, \; Im\Pi_{l,s} = \frac{1}{|\vec{q}|^2} Im\Pi_{2,s}, \tag{34}$$

$$\rho_{t,a} = \frac{-1}{2}(\rho_{1,a} + q^2 \rho_{l,a}), \; \rho_{t,s} = \frac{-1}{2}(\rho_{1,s} + q^2 \rho_{l,s}), \tag{35}$$

the annihilation and scattering parts of ρ_t at nonzero momentum is obtained as:

$$\rho_{t,a} = \frac{3q^2}{32\pi^2} \int_{-v}^{v} dx (2 - v^2 + x^2)[1 - 2n_+(x)], \tag{36}$$

$$\rho_{t,s} = -\frac{3q^2}{32\pi^2} \int_{v}^{\infty} dx (2 - v^2 + x^2)[n_-(x) - n_+(x)]. \tag{37}$$

The annihilation part of ρ_l, i.e., $\rho_{l,a}$ and its scattering part $\rho_{l,s}$ also at nonzero momentum can be found from Eqs. (28) and (29) replacing the coefficient $(2-\nu^2+x^2)$ by $2(1-x^2)$.
In our calculations, we also take into account the perturbative two-loop order α_s correction to the spectral density. This correction at zero temperature can be written as [1,3]:

$$\rho_{\alpha_s}(s) = \alpha_s \frac{s}{6\pi^2}\nu(s)\Big(3-\nu^2(s)\Big)\left[\frac{\pi}{2\nu(s)} - \frac{1}{4}\Big(3+\nu(s)\Big)\Big(\frac{\pi}{2}-\frac{3}{4\pi}\Big)\right], \qquad (38)$$

where, we replace the strong coupling α_s in Eq. (38) with its temperature dependent lattice improved expression $\alpha(T) = 2.095(82)\frac{g^2(T)}{4\pi}$ [16,22]and

$$g^{-2}(T) = \frac{11}{8\pi^2}\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right) + \frac{51}{88\pi^2}\ln\left[2\ln\left(\frac{2\pi T}{\Lambda_{\overline{MS}}}\right)\right]. \qquad (39)$$

where $\Lambda_{\overline{MS}} = T_c/1.14(4)$ and $T_c = 0.160 GeV$.

Similarly, the annihilation and scattering parts of spectral density for pseudo(scalar) particles is found as:

$$\rho_{a,pert}(s,T)=\rho_0(s)\Big[1-n\Big(\frac{\sqrt{s}}{2}\Big(1+\frac{m_1^2-m_2^2}{s}\Big)\Big)-n\Big(\frac{\sqrt{s}}{2}\Big(1-\frac{m_1^2-m_2^2}{s}\Big)\Big)\Big], \quad (40)$$

for $(m_1+m_2)^2 \leq s \leq \infty$,

$$\rho_{s,pert}(s,T)=\rho_0(s)\Big[n\Big(\frac{\sqrt{s}}{2}\Big(1+\frac{m_1^2-m_2^2}{s}\Big)\Big)-n\Big(-\frac{\sqrt{s}}{2}\Big(1-\frac{m_1^2-m_2^2}{s}\Big)\Big)\Big], \quad (41)$$

for $0 \leq s \leq (m_1-m_2)^2$, with $m_1 \geq m_2$. Here, $\rho_0(s)$ is the spectral density in the lowest order of perturbation theory at zero temperature and it is given by

$$\rho_0(s)=\frac{3}{8\pi^2 s}q^2(s)v^n(s), \quad (42)$$

where $q(s)=s-(m_1-m_2)^2$ and $v(s)=\Big(1-4m_1m_2/q(s)\Big)^{1/2}$. Here $n=3$ and $n=1$ for scalar and pseudoscalar particles, respectively. As it is seen, at $T\to 0$ limit these expressions are in good consistency with the vacuum expressions.

As an example, we present the dependence of the annihilation and scattering parts of the spectral density for $K^{\pm}$ and $D^{\pm}$ particles in Figs. 1 and 2. In numerical analysis, we use the values $m_s = 0,13$ GeV and $m_c = 1,46$ GeV for the quark masses. As it is clear, in the region of the standard threshold for particle decays, the $\rho_0(s)$ is replaced by the annihilation term. In the case of light mesons, the values of $\rho_{a,pert}(s, T)$ considerably differ from those of the $\rho_0(s)$. However, in the case of heavy mesons, the $\rho_{a,pert}(s, T)$ and $\rho_0(s)$ values are very close to each other. From Fig. 1, we also see that the in light $K^{\pm}$ cases, the medium contributions play important role and consist higher percentage of the total value.
Our concluding result is that the thermal contributions contribute significantly to the spectral function.

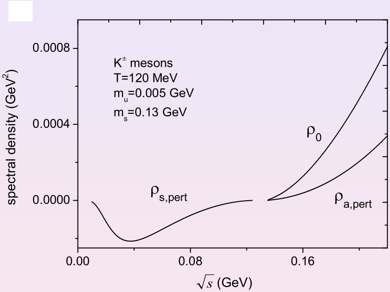

Figure: 1. The dependence of the spectral density of $K^{\pm}$ meson at temperature $T = 120$ MeV on the $\sqrt{s}$ parameter.

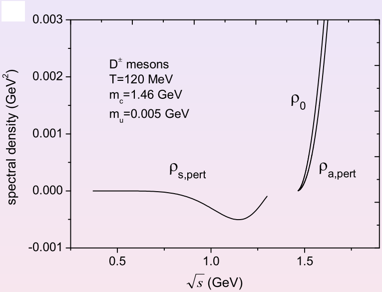

Figure: 1. The dependence of the spectral density of $D^{\pm}$ meson at temperature $T = 120$ MeV on the $\sqrt{s}$ parameter.

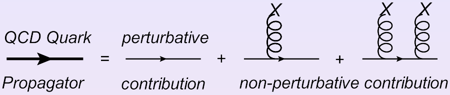

Figure: 1. Quark propagator.

Now, we proceed to calculate the nonperturbative part in QCD side. For this aim, we use the nonperturbative part of the quark propagator in an external gluon field, $A_\mu^a(x)$ in the Fock-Schwinger gauge, $x^\mu A_\mu^a(x) = 0$. Taking into account one and two gluon lines attached to the quark line, the massive quark propagator can be written in momentum space as [3]:

$$\begin{aligned} S^{aa'\,nonpert}(k) &= -\frac{i}{4} g (t^c)^{aa'} G^c_{\kappa\lambda}(0) \frac{1}{(k^2-m^2)^2} \left[\sigma_{\kappa\lambda}(\not{k}+m) + (\not{k}+m)\sigma_{\kappa\lambda} \right] \\ &- \frac{i}{4} g^2 (t^c t^d)^{aa'} G^c_{\alpha\beta}(0) G^d_{\mu\nu}(0) \frac{\not{k}+m}{(k^2-m^2)^5} \\ &\times (f_{\alpha\beta\mu\nu} + f_{\alpha\mu\beta\nu} + f_{\alpha\mu\nu\beta})(\not{k}+m), \end{aligned} \tag{43}$$

where,

$$f_{\alpha\beta\mu\nu} = \gamma_\alpha(\not{k} + m)\gamma_\beta(\not{k} + m)\gamma_\mu(\not{k} + m)\gamma_\nu. \tag{44}$$

We also need to know the expectation value $\langle TrG_{\alpha\beta}G_{\mu\nu}\rangle$. The Lorentz covariance at finite temperature permits us to write the general structure of this expectation value in the following manner:

$$\begin{aligned}\langle Tr^c G_{\alpha\beta}G_{\mu\nu}\rangle &= \frac{1}{24}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu})\langle G^a_{\lambda\sigma}G^{a\lambda\sigma}\rangle \\ &+ \frac{1}{6}\Big[g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu} - 2(u_\alpha u_\mu g_{\beta\nu} - u_\alpha u_\nu g_{\beta\mu} \\ &- u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu})\Big]\langle u^\lambda \Theta^g_{\lambda\sigma}u^\sigma\rangle,\end{aligned} \tag{45}$$

where, u^μ is the four-velocity of the heat bath and it is introduced to restore Lorentz invariance formally in the thermal field theory. In the rest frame of the medium $u^\mu = (1,0,0,0)$ and $u^2 = 1$ and $\Theta^g_{\lambda\sigma}$ is the traceless gluonic part of the enrgy - momentum tensor of the QCD.

Up to terms required for our calculations, the non perturbative part of massive quark propagator at finite temperature is obtained as:

$$
\begin{aligned}
S^{aa'\,nonpert}(k) &= -\frac{i}{4} g (t^c)^{aa'} G^c_{\kappa\lambda} \frac{1}{(k^2-m^2)^2} \Big[\sigma_{\kappa\lambda}(\not k + m) + (\not k + m)\sigma_{\kappa\lambda}\Big] \\
&+ \frac{i\, g^2\, \delta^{aa'}}{9\,(k^2-m^2)^4} \Big\{ \frac{3m(k^2 + m\not k)}{4} \langle G^c_{\alpha\beta} G^{c\alpha\beta}\rangle + \Big[m\Big(k^2 - 4(k\cdot u)^2\Big) \\
&+ \Big(m^2 - 4(k\cdot u)^2\Big)\not k + 4(k\cdot u)(k^2-m^2)\not u \Big] \langle u^\alpha \Theta^g_{\alpha\beta} u^\beta\rangle \Big\}. \qquad (46)
\end{aligned}
$$

Using the above expression and after straightforward but lengthy calculations, the nonperturbative part in QCD side is obtained as:

$$\begin{aligned}
\Pi_t^{nonpert} &= \int_0^1 dx \Big\{ -\frac{\langle \alpha_s G^2 \rangle}{72\pi\Big[m^2+q^2(-1+x)x\Big]^4}\Big[6q^6(-1+x)^4x^4+6m^2q^4x^2 \\
&\times (-1+x)^2(1-6x+6x^2)m^6(5-32x+42x^2-20x^3+10x^4) \\
&+ m^4q^2x\Big(-14+95x-140x^2+65^3+6x^4-2x^5\Big)\Big] \\
&- \frac{\alpha_s\langle u^\alpha \Theta^g_{\alpha\beta} u^\beta \rangle}{54\pi\Big[m^2+q^2(-1+x)x\Big]^4}\Big[x(-1+x)\Big(4q^4x^2(1-3x+2x^2)^2 \\
&+ m^4(12-35x+21x^2+28x^3-14x^4) \\
&+ m^2q^2x(-13+55x-82x^2 \\
&+ 36x^3+6x^4-2x^5)\Big)\Big(q^2-4(q\cdot u)^2\Big)\Big]\Big\}, \qquad (47)
\end{aligned}$$

where, $\langle G^2 \rangle = \langle G^c_{\alpha\beta} G^{c\alpha\beta} \rangle$.

In order to obtain QCD sum rules, we apply Borel transformation with respect to the $Q_0^2 = -q_0^2$ is applied to both sides of the sum rules for physical quantities.
To calculate the phenomenological part, we insert a complete set of intermediate states owing the same quantum numbers with current J between the currents in Eq. (1) and perform the integral over. As a result, at $T = 0$, we obtain

$$\Pi(q,0) = \frac{\langle 0 \mid J(0) \mid P\rangle\langle P \mid J(0) \mid 0\rangle}{m_P^2 - q^2} \quad + \quad \cdots, \tag{48}$$

where $\cdots$ represents the contributions of the higher and continuum states , P indicates pseudo (scalar) and vector mesons and m_P is mass of considering particle. The decay constants pseudo(scalar) and vector meson are defined by the matrix element of the pseudo(scalar) and vector current $J(0)$ between the vacuum and hadronic states:

$$\langle 0|J(0)|P\rangle = Af_P m_P \tag{49}$$

where $A = 1$ for pseudo(scalar) particles, respectively and $A = \varepsilon_\mu^\lambda$ as for vector particles. Here ε_μ^λ is polarization states of vector mesons. Note that Eqs. (3) and (4) are valid also at finite temperature, hence, the final representation for the physical side can be written in terms of the temperature dependent mass and decay constant as:

$$\Pi(q,T) = \frac{f_P^2(T)m_P^2(T)}{m_P^2(T) - q^2} \quad + \quad \cdots \tag{50}$$

In order to obtain thermal sum rules, now we equate the spectral representation and results of operator product expansion for amplitudes $\Pi_l(q^2,\omega)$ or $\Pi_t(q^2,\omega)$ at sufficiently high Q_0^2. When performing numerical results, we should exchange our reference to one at which the particle is at rest, i.e., we shall set $|\mathbf{q}| \to 0$. In this limit since the functions Π_l and Π_t are related to each other, it is enough to use one of them to acquire thermal sum rules. Here, we use the function Π_t. Equating the OPE and hadronic representations of the correlation function and applying quark-hadron duality, our sum-rule takes the form:

$$\frac{f_V^2 Q_0^4}{(m_V^2 + Q_0^2)\; m_V^2} = Q_0^4 \int_{4m^2}^{s_0} \frac{[\rho_{t,a}(s) + \rho_{\alpha_s}(s)]}{s^2(s+Q_0^2)} ds + \int_0^{|\mathbf{q}|^2} \frac{\rho_{t,s}}{s+Q_0^2} ds + \Pi_t^{nonpert}, \qquad (51)$$

where, for simplicity, the total decay width of meson has been neglected. In derivation of Eq.(**??**) we have also used summation over polarization states, $\sum_\lambda \varepsilon_\mu^{(\lambda)*} \varepsilon_\nu^{(\lambda)} = -(g^{\mu\nu} - q_\mu q_\nu / m_V^2)$. The Borel transformation removes subtraction terms in the dispersion relation and also exponentially suppresses the contributions coming from the excited resonances and continuum states heavier than considered vector ground states.

Applying Borel transformation with respect to Q_0^2 to both sides of Eq.(51), we obtain

$$\begin{aligned} f_V^2 m_V^2 \exp\left(-\frac{m_V^2}{M^2}\right) &= \int_{4m^2}^{s_0} ds\, [\rho_{t,a}(s)+\rho_{\alpha_s}(s)] e^{-\frac{s}{M^2}} \\ &+ \int_0^{|\mathbf{q}|^2} ds\, \rho_{t,s}(s) e^{-\frac{s}{M^2}} + \widehat{B}\Pi_t^{nonpert}. \end{aligned} \tag{52}$$

As we also previously mentioned, when doing numerical analysis, we will set $|\mathbf{q}| \rightarrow 0$ representing the rest frame of the particle. In this case, the scattering cut shrinks to a point and the spectral density becomes a singular function. Hence, the second term in the right side of Eq.(52) must be detailed analyzed. Detailed analysis shows that

$$\lim_{|\mathbf{q}|\rightarrow 0} \int_0^{|\mathbf{q}|^2} ds \rho_{t,s}(s) exp\left(-\frac{s}{M^2}\right) = 0. \tag{53}$$

In Eq.(52), $\widehat{B}\Pi_t^{nonpert}$ shows the nonperturbative part of QCD side in Borel transformed scheme, which is given by:

$$\begin{aligned}
\hat{B}\Pi_t^{nonpert} &= \int_0^1 dx \, \frac{1}{144\,\pi\, M^6\, x^4\,(-1+x)^4} \exp\left[\frac{m^2}{M^2\, x\,(-1+x)}\right]\Big\{\langle\alpha_s G^2\rangle \\
\times & \Big[12\, M^6\, x^4\,(-1+x)^4 - m^6\,(1-2x)^2(-1-x+x^2) - 12\, m^2\, M^4\, x^2\,(-1+x)^2 \\
\times & (1-3x+3x^2) + m^4\, M^2\, x\,(-2+19x-32x^2+11x^3+6x^4-2x^5)\Big] + 4\,\alpha_s\langle\Theta^g\rangle \\
\times & \Big[-8\, M^6\, x^3\,(1-2x)^2(-1+x)^3 + m^6\,(1-2x)^2(-1-x+x^2) - 2\, m^2\, M^4\, x^2 \\
\times & (-1+x)^2(-1-6x+8x^2-4x^3+2x^4) + m^4\, M^2\, x\,(-2+3x-12x^2 \\
+ & 31x^3-30x^4+10x^5)\Big]\Big\}, \qquad (54)
\end{aligned}$$

where, $\Theta^g = \Theta^g_{00}$.

In this section, we discuss the temperature dependence of the masses and leptonic decay constants of the J/ψ and Υ vector mesons. Taking into account the above Eqs. and applying derivative with respect to $1/M^2$ to both sides of the Eq.(52) and dividing by themselves, we obtain

$$m_V^2(T) = \frac{\int_{4m^2}^{s_0(T)} ds\, s\, [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \Pi_1^{nonpert}(M^2, T)}{\int_{4m^2}^{s_0(T)} ds[\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \widehat{B}\Pi_t^{nonpert}}, \quad (55)$$

and

$$f_V^2(T) = \frac{1}{m_V^2(T)}\left(\int_{4m^2}^{s_0} ds\, [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] e^{-\frac{s}{M^2}} + \widehat{B}\Pi_t^{nonpert}\right) \exp\left(\frac{m_V^2}{M^2}\right). \quad (56)$$

where

$$\Pi_1^{nonpert}(M^2, T) = M^4 \frac{d}{dM^2} \widehat{B}\Pi_t^{nonpert}, \quad (57)$$

and

$$\rho_{t,a}(s) = \frac{1}{8\pi^2} s\nu(s)(3 - \nu^2(s)) \left[1 - 2n\left(\frac{\sqrt{s}}{2}\right)\right]. \quad (58)$$

The hadronic spectral density is expressed by the ground state pseudoscalar meson pole plus the contribution of the higher states and continuum:

$$\rho^{had}(s) = \frac{f_{B_c}^2(T) m_{B_c}^4(T)}{(m_b + m_c)^2}\delta(s - m_{B_c}^2) + \theta(s - s_0)\rho^{pert}(s) \quad (59)$$

Matching the phenomenological and QCD sides of the correlation function, sum rules for the mass and decay constant of pseudoscalar meson are obtained. Performing Borel transformation over the $Q_0^2 = -q_o^2$ after lengthy calculations, we obtain the following sum rule for B_c mesons:

$$\begin{aligned} f_{B_c}^2(T)\, m_{B_c}^4(T)\, exp\Big(-\frac{m_{B_c}^2}{M^2}\Big) &= (m_b + m_c)^2 \\ \times\Big\{ \int_{(m_b+m_c)^2}^{s_0(T)} ds\, (\rho^{a,pert}(s) + \rho_{\alpha_s}(s))\, exp\Big(-\frac{s}{M^2}\Big) \\ + \int_0^{(m_b-m_c)^2} ds\, \rho^{s,pert}(s)\, exp\Big(-\frac{s}{M^2}\Big) \\ + \widehat{B}\Pi^{nonpert}\Big\}, \end{aligned} \quad (60)$$

where M^2 is the Borel mass parameter and $\hat{B}\Pi^{nonpert}$ shows the nonperturbative part of QCD side in Borel transformed scheme:

$$\hat{B}\Pi^{nonpert} = \int_0^1 dx\, \frac{1}{96\,\pi\, M^6\, x^4\,(-1+x)^4}$$
$$\times \exp\left[\frac{m_c^2 x - m_b^2(-1+x)}{M^2 x(-1+x)}\right]\Big\{\langle\alpha_s G^2\rangle\Big[-m_b^6(-1+x)^6$$
$$+m_b^5 m_c(-1+x)^4\, x(-1+2x) + x^4\Big(-12\, m_c^2\, M^4$$
$$\times(-1+x)^3 + 12\, M^6\,(-1+x)^4 + 2\, m_c^4\, M^2\, x\,(-1+x)$$
$$-m_c^6 x^2\Big) + m_b^4 x(-1+x)^3\Big(2M^2(-1+x)^2 + m_c^2$$
$$\times(1-3x+x^2)\Big) + m_b^2 x^2(-1+x)\Big(12M^4 x(-1+x)^3$$
$$+m_c^4 x(-1+x+x^2) + 3m_c^2 M^2(1-3x+4x^2-2x^3)\Big)$$
$$+m_b^3 m_c x(-1+x)^2\Big(-m_c^2 x(1-2x)^2 + M^2(2-9x$$
$$+6x^2+x^3)\Big) - m_b m_c(-1+x)x^2\Big(m_c^4 x^2(1-2x)$$
$$-m_c^2 M^2 x(6-9x+x^2) + 6M^4(1+x-4x^2+2x^3)\Big)\Big] \qquad (61)$$

$$
\begin{aligned}
&+3\,\alpha_s\langle\Theta^g\rangle\Big[m_b^6(-1+x)^6-m_b^5m_cx(-1+x)^4(-1+2x)\\
&+m_b\,m_cx^3(-1+x)\Big(m_c^4x(1-2x)+4\,M^4\,(-1+x)^2\\
&\times(2-x+x^2)+m_c^2\,M^2\,(-4+3x+5x^2-4x^3)\Big)\\
&-m_b^4x(-1+x)^3\Big(m_c^2(1-3x+x^2)+2M^2(1-2x+x^3)\Big)\\
&-m_b^2x^2(-1+x)\Big(m_c^4x(-1+x+x^2)+m_c^2M^2\\
&\times(5-17x+24x^2-12x^3)+M^4(-1+x)^2(-1+15x\\
&-7x^2+2x^3)\Big)+x^3\Big(m_c^6x^3+M^6(-1+x)^3(9-11x\\
&+11x^2)+2m_c^4M^2x(-1+4x-4x^2+x^3)-m_c^2M^4\\
&\times(-1+x)^2(-9+7x+x^2+2x^3)\Big)+m_b^3m_cx^2(-1+x)^2\\
&\times\Big(m_c^2(1-2x)^2+M^2(1+6x-11x^2+4x^3)\Big)\Big]\Big\}. \qquad (62)
\end{aligned}
$$

We use the gluonic part of the energy density both obtained from lattice QCD and chiral perturbation theory. In the rest frame of the heat bath, the total energy density obtained using lattice QCD is well fitted by the help of the following parametrization:

$$\langle \Theta \rangle = 2\langle \Theta^g \rangle = 6 \times 10^{-6} exp[80(T - 0.1)](GeV^4), \quad (63)$$

where temperature T is measured in units of GeV and this parametrization is valid only in the region $0.1\ GeV \leq T \leq 0.17\ GeV$. Here, we should stress that the total energy density has been calculated for $T \geq 0$ in chiral perturbation theory, while this quantity has only been obtained for $T \geq 100\ MeV$ in lattice QCD.

In low temperature chiral perturbation limit, the thermal average of the energy density is expressed as :

$$\langle\Theta\rangle = \langle\Theta^{\mu}_{\mu}\rangle + 3\,p, \tag{64}$$

where $\langle\Theta^{\mu}_{\mu}\rangle$ is trace of the total energy momentum tensor and p is pressure. These quantities are given by:

$$\begin{aligned} \langle\Theta^{\mu}_{\mu}\rangle &= \frac{\pi^2}{270}\frac{T^8}{F_\pi^4}\ln\Big(\frac{\Lambda_p}{T}\Big), \\ p &= 3T\Big(\frac{m_\pi\;T}{2\;\pi}\Big)^{\frac{3}{2}}\Big(1+\frac{15\;T}{8\;m_\pi}+\frac{105\;T^2}{128\;m_\pi^2}\Big)exp\Big(-\frac{m_\pi}{T}\Big), \end{aligned} \tag{65}$$

where $\Lambda_p = 0.275\;GeV$, $F_\pi = 0.093\;GeV$ and $m_\pi = 0.14\;GeV$.

We use the temperature dependent continuum threshold $s_0(T)$ and gluon condensate$\langle G^2 \rangle$ in the following form :

$$s_0(T) = s_0 \left[1 - \left(\frac{T}{T_c^*} \right)^8 \right] + 4\ m_Q^2 \ \left(\frac{T}{T_c^*} \right)^8, \tag{66}$$

where $T_c^* = 1.1 \times T_c = 0.176\ GeV$.

$$\langle G^2 \rangle = \frac{\langle 0|G^2|0\rangle}{exp\left[12\left(\frac{T}{T_c} - 1.05 \right) \right] + 1}. \tag{67}$$

In further analysis, we use the values, $m_c = (1.3 \pm 0.05)\ GeV$, $m_b = (4.7 \pm 0.1)\ GeV$ and $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0\rangle = (0.012 \pm 0.004)\ GeV^4$ for quarks masses and gluon condensate at zero temperature. The sum rules for the masses and decay constants also include two parametres : continuum threshold s_0 and Borel mass parameter M^2. The continuum threshold, s_0 is not completely arbitrary and it is related to the energy of the first exited state with the same quantum numbers as the interpolating currents. Our numerical analysis show that in the intervals $s_0 = (11 - 13)\ GeV^2$ and $s_0 = (98 - 102)\ GeV^2$, respectively for the J/ψ and Υ channels, the results weakly depend on this parameter. The working region for the Borel mass parameter, M^2 is determined demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from the higher dimensional operators are small. As a result, the working region for the Borel parameter is found to be $8\ GeV^2 \leq M^2 \leq 25\ GeV^2$ and $12\ GeV^2 \leq M^2 \leq 35\ GeV^2$ in J/ψ and Υ channels, respectively.

Our final task is to discuss the temperature dependence of the leptonic decay constant of the considered particles. For this aim, we plot these quantities in terms of temperature in figures [2-5] using the total energy density from both chiral perturbation theory and lattice QCD and at different values of the s_0 but a fixed value of the Borel mass parameter. As shown in this graphs, at $T = 0$, the values of the decay constants of the J/ψ and Υ are obtained as $f_{J/\psi} = (0.460 \pm 0.022)\ GeV$ and $f_{\Upsilon} = (0.715 \pm 0.032)\ GeV$. These results are in good consistency with the existing experimental data and predictions of the other nonperturbative models. Also, we observe that the decay constants remain insensitive to the variation of the temperature up to $T \cong 100\ MeV$, however after this point, they start to diminish increasing the temperature.At deconfinement or critical temperature, the decay constants approach roughly to 50% of their values at zero temperature, while the masses are decreased about 12%, and 2.5% for J/ψ and Υ states, respectively.

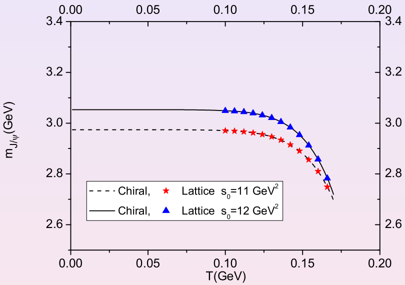

Figure: The dependence of the mass of J/ψ meson in vacuum on the Borel parameter M^2.

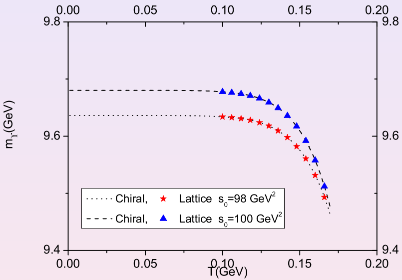

Figure: The dependence of the mass of Υ meson in vacuum on the Borel parameter M^2.

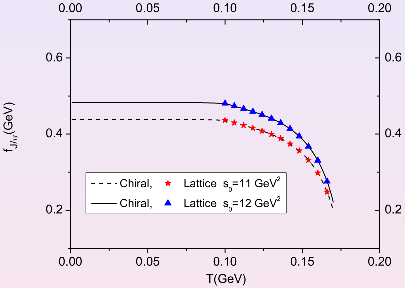

Figure: The dependence of the leptonic decay constant of J/ψ vector meson in GeV on temperature at $M^2 = 10\ GeV^2$.

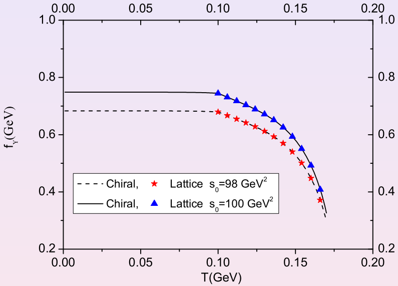

Figure: The dependence of the leptonic decay constant of Υ vector meson in GeV on temperature at $M^2 = 20\ GeV^2$.

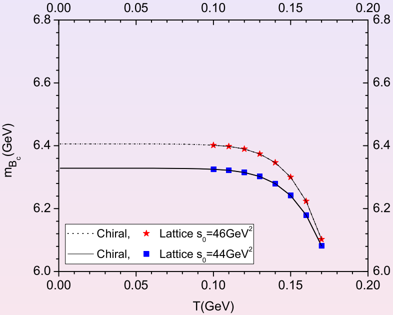

Figure: The dependence of the mass of B_c meson in GeV on temperature at $M^2 = 20\ GeV^2$.

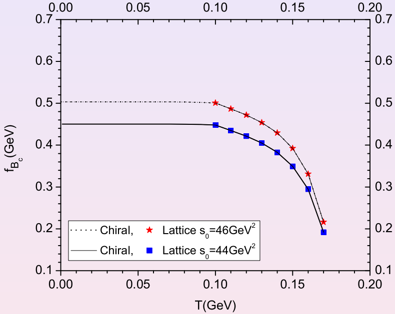

Figure: The dependence of the leptonic decay constant of B_c meson in GeV on temperature at $M^2 = 20\ GeV^2$.

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THANKS !

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