Massive Higher Derivative Gravity Theories

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Based On

This thesis is based on the papers

- I. G., B. Tekin, *"Massive Higher Derivative Gravity in D-dimensional Anti-de Sitter Spacetimes"*, P. R. D **80**, 064033 (2009),

and

- I. G., T. Ç. Şişman, B. Tekin, *"Canonical Strucfture of Higher Derivative Gravity in 3D"*, P. R. D **80**, 064033 (2009) D **81**, 104017 (2010).

Introduction

- Experience from quantum field theory implies that at high energies Einstein's gravity should be replaced with: Einstein-Hilbert term+Higher Curvature terms,
- Higher curvature terms are motivated by the quantum gravity scenarios such as string theory and asymptotic safety,
- To have a better IR behaviour a mass term can be added to the theory,
- Mass can be given to graviton by adding a Pauli-Fierz mass term,
- In the first part the general quadratic massive gravity theory will be analyzed,
- In the second part the canonical structure of the general quadratic action will be discussed.

Tree-Level Unitarity

- To have a physically meaningful theory it must be unitary.
- Tree level unitarity is tachyon and ghost freedom.
  - Ghost is characterized by negative kinetic energy,
  - Tachyon is characterized by negative mass square.
- Thus, unitarity analysis is basically a check of proper signs in the graviton propagator $(-,+,+,\dots)$ that is $\frac{1}{p^2+m^2}$.
- In the canonical form $(\Box - m^2)\psi = 0,\ m^2 > 0$.

Equations of Motion

- The most general quadratic gravity model augmented with a Pauli-Fierz mass term is

$$
\begin{aligned}
I &= \int d^D x \sqrt{-g} \left\{ \frac{1}{\kappa} (R - 2\Lambda_0) + \alpha R^2 + \beta R^2_{\mu\nu} + \gamma \left(R^2_{\mu\nu\sigma\rho} - 4R^2_{\mu\nu} + R^2 \right) \right\} \\
&+ \int d^D x \sqrt{-g} \left\{ -\frac{M^2}{4\kappa} \left(h^2_{\mu\nu} - h^2 \right) + \mathcal{L}_{\text{matter}} \right\} , \qquad (1)
\end{aligned}
$$

- To get the one-particle exchange amplitude we need the linear equations of motion:
  - we take the variation of (1) with respect to the metric $g_{\mu\nu}$, $(-,+,+,\dots)$ to get the equations of motion,
  - then we linearize the equations of motion around a constant curvature background $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- The linearized equations of motion are

$$
\begin{aligned}
T_{\mu\nu}(h) &= a\mathcal{G}^L_{\mu\nu} + (2\alpha + \beta) \left(\bar{g}_{\mu\nu} \bar{\Box} - \bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{2\Lambda}{D-2} \bar{g}_{\mu\nu} \right) R^L \\
&+ \beta \left(\bar{\Box} \mathcal{G}^L_{\mu\nu} - \frac{2\Lambda}{D-1} \bar{g}_{\mu\nu} R^L \right) + \frac{M^2}{2\kappa} \left(h_{\mu\nu} - \bar{g}_{\mu\nu} h \right) , \qquad (2)
\end{aligned}
$$

where we have defined $a \equiv \frac{1}{\kappa} + \frac{4\Lambda D}{D-2}\alpha + \frac{4\Lambda}{D-1}\beta + \frac{4\Lambda(D-3)(D-4)}{(D-1)(D-2)}\gamma$.

Tree-Level Scattering Amplitude

- To get the physical parts of $h_{\mu\nu}$ we decompose it as

$$h_{\mu\nu} \equiv h_{\mu\nu}^{TT} + \bar{\nabla}_{(\mu} V_{\nu)} + \bar{\nabla}_{\mu} \bar{\nabla}_{\nu} \phi + \bar{g}_{\mu\nu} \psi, \tag{3}$$

- Taking divergence and double divergence of (3)

$$h = \bar{\square}\phi + D\psi, \qquad \bar{\square}h = \bar{\square}^2\phi + \frac{2\Lambda}{(D-2)}\bar{\square}\phi + \bar{\square}\psi, \tag{4}$$

  where we used $\bar{\nabla}^{\nu}\bar{\nabla}^{\mu}h_{\mu\nu} = \bar{\square}h$, which is *not* a gauge condition but imposed on us as a result of the nonzero mass term.
- Using (4)

$$\psi = \left\{ \frac{\Lambda}{\kappa} + 4\Lambda f - c\Lambda\bar{\square} - \frac{M^2}{2\kappa}(D-1) \right\}^{-1} \left(\frac{(D-1)(D-2)}{2\Lambda}\bar{\square} + D \right)^{-1} T, \tag{5}$$

  where $c \equiv \frac{4(D-1)\alpha}{D-2} + \frac{D\beta}{D-2}$.
- Decomposing the energy-momentum tensor one can write the one-particle exchange amplitude between two covariantly conserved sources as

$$A = \frac{1}{4}\int d^D x\sqrt{-\bar{g}}\, T'_{\mu\nu}(x)\, h^{\mu\nu}(x) = \frac{1}{4}\int d^D x\sqrt{-\bar{g}} \left(T'_{\mu\nu} h^{TT\mu\nu} + T'\psi \right).$$

Tree-Level Scattering Amplitude

- Finally,

$$\begin{aligned}
4A &= 2T'_{\mu\nu}\left\{(\beta\bar{\Box}+a)(\triangle_L^{(2)}-\frac{4\Lambda}{D-2})+\frac{M^2}{\kappa}\right\}^{-1}T^{\mu\nu} \\
&+ \frac{2}{D-1}T'\left\{(\beta\bar{\Box}+a)(\bar{\Box}+\frac{4\Lambda}{D-2})-\frac{M^2}{\kappa}\right\}^{-1}T \\
&- \frac{4\Lambda}{(D-2)(D-1)^2}T'\left\{(\beta\bar{\Box}+a)(\bar{\Box}+\frac{4\Lambda}{D-2})-\frac{M^2}{\kappa}\right\}^{-1}\left\{\bar{\Box}+\frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T \\
&+ \frac{2}{(D-2)(D-1)}T'\left\{\frac{1}{\kappa}+4\Lambda f-c\bar{\Box}-\frac{M^2}{2\kappa\Lambda}(D-1)\right\}^{-1}\left\{\bar{\Box}+\frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T.
\end{aligned} \tag{7}$$

where $f \equiv (\alpha D+\beta)\frac{(D-4)}{(D-2)^2}+\gamma\frac{(D-3)(D-4)}{(D-1)(D-2)}$.

Tree-Level Scattering Amplitude

- From (7) one can figure out the particle spectrum,
- One can also compute the Newtonian potentials in flat spacetime,
- In curved background the Green's function (matrix) of Lichnerowicz operator must be handled,
- We can see the $M^2 \to 0$ and $\Lambda \to 0$ limits does not commute,
- First taking the flatspace limit we encounter the van Dam-Veltman-Zakharov (vDVZ) discontinuity,
- First taking the massless limit take us to New Massive Gravity (NMG) theory.

vDVZ Discontinuity

- For this case the amplitude become

$$4A = -2T'_{\mu\nu}\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2 - \frac{M^2}{\kappa}\right\}^{-1}T^{\mu\nu} + \frac{2}{D-1}T'\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2 - \frac{M^2}{\kappa}\right\}^{-1}T \quad (8)$$

Unless $\beta = 0$, we have a massive ghost.

- The Newtonian potential energy between $T'_{00} \equiv m_1\delta(x - x_1)$, $T^{00} \equiv m_2\delta(x - x_2)$ in three and four dimensions can be obtained as

$$\begin{aligned} U &= \frac{1}{2\beta(m_+^2 - m_-^2)}\frac{m_1 m_2}{4\pi}[K_0(m_- r) - K_0(m_+ r)] \; D = 3, \\ U &= \frac{m_1 m_2}{3\beta(m_+^2 - m_-^2)}\frac{1}{4\pi r}[e^{-m_- r} - e^{-m_+ r}] \; D = 4. \end{aligned} \quad (9)$$

where $r \equiv |\vec{x_1} - \vec{x_2}|$.

- As $\beta \to 0$, the potential energies become

$$U = -\frac{\kappa}{8\pi}m_1 m_2 K_0(Mr) \qquad D = 3, \quad (10)$$

$$U = -\frac{4}{3}\frac{Gm_1 m_2}{r}e^{-Mr} \qquad D = 4 \quad (11)$$

here K_0 is the modified Bessel function of the second kind.

New Massive Gravity

- For $M^2 = 0$ then taking $\Lambda \to 0$ limit

$$4A = -2T'_{\mu\nu}\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2\right\}^{-1} T^{\mu\nu} + \frac{2}{(D-1)} T'\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2\right\}^{-1} T - \frac{2}{(D-1)(D-2)} T'\left\{c\partial^4 - \frac{1}{\kappa}\partial^2\right\}^{-1} T \qquad (12)$$

- Generically there are three poles :

$$\partial_1^2 = 0, \qquad \partial_2^2 = -\frac{1}{\kappa\beta} \qquad , \partial_3^2 = \frac{1}{\kappa c}. \qquad (13)$$

$$Res(\partial_1^2) = \frac{2\kappa\,(3-D)}{(D-2)}, \; Res(\partial_2^2) = \frac{2\kappa\,(D-2)}{(D-1)}, \; Res\,(\partial_3^2) = -\frac{2\kappa}{(D-1)\,(D-2)}$$

- From the second pole and its residue; $\kappa\beta < 0$ and $\kappa < 0$,
- From the residue of the massless pole; $D = 3$
- The residue of the third pole becomes positive for negative κ. To eliminate this residue $c = 8\alpha + 3\beta = 0$.

New Massive Gravity

- Newtonian potential

$$U = \frac{\kappa}{8\pi} m_1 m_2 \left(K_0(m_g r) - K_0(m_0 r) \right) \qquad D = 3, \qquad (14)$$

  where $m_g^2 \equiv -\frac{1}{\kappa\beta}$ and $m_0^2 \equiv \frac{1}{\kappa(8\alpha+3\beta)}$. Clearly, m_0 is a massive ghost that gives a repulsive component.

- This result also confirms that, at this level, NGM has the same Newtonian limit as the usual massive gravity (10), if the Pauli-Fierz mass term is chosen as $M = m_g$.
- Beyond three dimensions, in flat space, massive ghost does not decouple unless $\beta = 0$. As an example, let us look at $D = 4$:

$$U = -\frac{G m_1 m_2}{r} \left(1 - \frac{4}{3} e^{-m_g r} + \frac{1}{3} e^{-m_a r} \right), \qquad (15)$$

  where $m_a^2 \equiv \frac{1}{2\kappa(3\alpha+\beta)}$. The middle, repulsive term signals the ghost problem. K.S. Stelle, P. R. D **16**, 953 (1977).

Non-Linear Action

- Generic action in non-linear level is

$$I = \int d^3x \sqrt{-g} \left[\frac{1}{\kappa} R + \alpha R^2 + \beta R_{\mu\nu}^2 + \mathcal{L}_{matter} \right] \tag{16}$$

  - κ, α, β are coupling constants whose signs will be fixed later,
  - $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, here $\eta_{\mu\nu}$ is the usual flat space-time metric.
- Our signature is (-,+,+).
- In this conventions the Klein-Gordon equation reads $(\Box - m^2)\psi = 0$,
  - where $\Box = \partial_\mu \partial^\mu = -\partial_0^2 + \nabla^2$.

Linear Action

- The action for $h_{\mu\nu}$ is

$$I=-\frac{1}{2}\int d^3x\, h_{\mu\nu}\left[\frac{1}{\kappa}\mathcal{G}_L^{\mu\nu}+(2\alpha+\beta)(\eta^{\mu\nu}\Box-\partial^\mu\partial^\nu)R_L+\beta\Box\mathcal{G}_L^{\mu\nu}\right] \tag{17}$$

  - Here $\mathcal{G}_L^{\mu\nu}=R^L_{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}R_L$ is the linear Einstein tensor in the flat space-time,
  - where $R_L^{\mu\nu}=\frac{1}{2}\left(\partial_\sigma\partial^\mu h^{\nu\sigma}+\partial_\sigma\partial^\nu h^{\mu\sigma}-\Box h^{\mu\nu}-\partial^\mu\partial^\nu h\right)$ and $R_L=\partial_\alpha\partial_\beta h^{\alpha\beta}-\Box h$, and $h=\eta_{\mu\nu}h^{\mu\nu}$.

- Decomposition of $h_{\mu\nu}$ gives six free functions of space and time that are;
  - $h_{ij}=(\delta_{ij}+\hat{\partial}_i\hat{\partial}_j)\phi-\hat{\partial}_i\hat{\partial}_j\chi+(\epsilon_{ik}\hat{\partial}_k\hat{\partial}_j+\epsilon_{jk}\hat{\partial}_k\hat{\partial}_i)\xi$
  - $h_{0i}=-\epsilon_{ij}\partial_j\eta+\partial_i N_L$ and $h_{00}=N$ where $\hat{\partial}\equiv\partial_i/\sqrt{-\nabla^2}$.

Linear Action

- With these equations one can write the linearized Einstein tensor in terms of three functions (using Bianchi identity $\nabla^{\mu}\mathcal{G}_{\nu\mu}=0$ one can eliminate three of them) which are invariant under gauge transformations $\delta_{\zeta}h_{\mu\nu}=\partial_{\mu}\zeta_{\nu}+\partial_{\nu}\zeta_{\mu}$:
  - $\mathcal{G}^{L}_{00}=-\frac{1}{2}\nabla^{2}\phi$, $\mathcal{G}^{L}_{0i}=-\frac{1}{2}(\epsilon_{ik}\partial_{k}\sigma+\partial_{i}\dot{\phi})$ and
  - $\mathcal{G}^{L}_{ij}=-\frac{1}{2}\left[(\delta_{ij}+\hat{\partial}_{i}\hat{\partial}_{j})q-\hat{\partial}_{i}\hat{\partial}_{j}\ddot{\phi}-(\epsilon_{ik}\hat{\partial}_{k}\hat{\partial}_{j}+\epsilon_{jk}\hat{\partial}_{k}\hat{\partial}_{i})\dot{\sigma}\right]$.
  - Where $q\equiv\nabla^{2}N-2\nabla^{2}\dot{N}_{L}+\ddot{\chi}$, $\sigma\equiv\dot{\xi}-\nabla^{2}\eta$.

Canonical Form of The Linear Action

- After writing all the terms in the linear action in terms of the gauge invariant functions one get the canonical form as

$$\begin{aligned} I &= \frac{1}{2}\int d^3x\left[\frac{1}{\kappa}\phi q + (2\alpha+\beta)(q-\Box\phi)^2 + \beta q\Box\phi\right] \\ &\quad +\frac{\beta}{2}\int d^3x\left(\sigma\Box\sigma + \frac{1}{\kappa\beta}\sigma^2\right), \end{aligned} \tag{18}$$

- σ is a scalar field with mass $m_g^2 = -\frac{1}{\kappa\beta}$,
- To be tachyon-free $\kappa\beta < 0$,
- To be a non-ghost $\beta > 0$, therefore $\kappa < 0$.
- Let us analyze the two cases separately which are
  1. $2\alpha + \beta \neq 0$
  2. $2\alpha + \beta = 0$

$2\alpha+\beta\neq 0$ and $2\alpha+\beta=0$ Theories

- Taking variation with respect to q field and eliminating it one gets

$$I = \frac{1}{2}\int d^3x\left[\frac{\beta\left(8\alpha+3\beta\right)}{4\left(2\alpha+\beta\right)}\left(\Box\phi\right)^2+\frac{\left(4\alpha+\beta\right)}{2\kappa\left(2\alpha+\beta\right)}\phi\Box\phi-\frac{1}{4\kappa^2\left(2\alpha+\beta\right)}\phi^2\right] +\frac{\beta}{2}\int d^3x\left(\sigma\Box\sigma+\frac{1}{\kappa\beta}\sigma^2\right), \quad (19)$$

- $8\alpha+3\beta=0$: The higher derivative term vanishes and the theory does not describe a higher derivative Pais-Uhlenbeck oscillator

$$I_{NMG} = -\frac{1}{2\kappa}\int d^3x\left(\phi\Box\phi+\frac{1}{\kappa\beta}\phi^2\right)+\frac{\beta}{2}\int d^3x\left(\sigma\Box\sigma+\frac{1}{\kappa\beta}\sigma^2\right) \quad (20)$$

The NMG theory gives two massive spin-2 fields with the same masses therefore this is a parity invariant theory.

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- $2\alpha+\beta=0$: In this case the action is

$$I=\frac{m_g^2\beta}{2}\int d^3x\left[\left(\Psi_1\Box\Psi_1-m_g^2\Psi_1^2\right)-\left(\Psi_2\Box\Psi_2-m_g^2\Psi_2^2\right)\right]. \quad (21)$$

$2\alpha+\beta\neq 0$ and $2\alpha+\beta=0$ Theories

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Higher Derivative Gravity Plus Chern-Simons Term

- The gravitational Chern-Simons term is

$$-\frac{1}{2\mu}\int d^3x\,\sqrt{-g}\epsilon^{\lambda\mu\nu}\Gamma^{\rho}{}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}{}_{\rho\nu}+\frac{2}{3}\Gamma^{\sigma}{}_{\mu\beta}\Gamma^{\beta}{}_{\nu\rho}\right) \tag{22}$$

.

- Linearization of this term gives $I_{CS}=\frac{1}{2\mu}\int d^3x\,\sigma\left(q+\Box\phi\right)$, and the total action becomes

$$I=\frac{1}{2}\int d^3x\left[\frac{1}{\kappa}\left(\phi q+\sigma^2\right)+\left(2\alpha+\beta\right)\left(q-\Box\phi\right)^2 +\beta\left(q\Box\phi+\sigma\Box\sigma\right)+\frac{1}{\mu}\sigma\left(q+\Box\phi\right)\right] \tag{23}$$

  - Assuming $2\alpha+\beta\neq 0$,
  - Considering only the NMG limit
  - Eliminating the q term one gets

$$I_{NMG-CS}=\frac{\beta}{2}\int d^3x\left\{\left[\sigma\Box\sigma-\left(m_g^2+\frac{1}{\mu^2\beta^2}\right)\sigma^2\right]+\frac{2m_g^2}{\beta\mu}\sigma\phi +m_g^2\left(\phi\Box\phi-m_g^2\phi^2\right)\right\} \tag{24}$$

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Decoupling of σ and ϕ

- First take the Fourier transform of the fields
- Put the Lagrangian in matrix form
- Diagonalize the matrix

$$I_{BHT-CS} = \frac{\beta}{2} \int d^3x \left(\Psi_+ \Box \Psi_+ - m_+^2 \Psi_+^2 + \Psi_- \Box \Psi_- - m_-^2 \Psi_-^2 \right), \quad (25)$$

where $m_\pm^2 = m_g^2 + \frac{1}{2\mu^2\beta^2} \pm \frac{1}{\mu\beta}\sqrt{m_g^2 + \frac{1}{4\mu^2\beta^2}}$,

- Since $m_+ \neq m_-$ it is a parity violating theory
- $\beta \to 0$ gives a single degree of freedom where m_+ diverges and $m_- = -|\mu|/\kappa$.

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Conclusion

- We compute the one-particle scattering amplitude of the most general quadratic curvature gravity augmented with a PF mass term,
- For the flatspace and massless limit we encounter with the vDVZ discontinuity and NMG,
- NMG is non-ghost and tachyon-free theory at tree-level,
- The canonical structure of the linearized quadratic gravity models has been analyzed, in the flat spacetime,
- The analysis is done in a gauge invariant way,
- The general action is decoupled into three harmonic oscillators and only the NMG limit gives us a unitary and tachyon free theory,
- From the general canonical action one can see that NMG is not a higher derivative Pais-Uhlenbeck oscillator,
- We extend our discussion by adding a gravitational Chern-Simons term and analyze it for the NMG case, in the flat spacetime,
- The unitarity of NMG must be checked for loop levels.

The End

Thank You...