Properties of Light and Heavy Baryons in Light Cone QCD Sum Rules

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Outline

- Standard Model: electroweak and strong interactions
- strong interactions (QCD)
- non-perturbative methods
- QCD sum rules

Light baryons

- Classification: In SU(3) flavor symmetry: $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$
- Light decuplet baryons: (Spin 3/2)

$$
\begin{array}{lcl cccccccc}
s & = & 0 & \Delta^{-} & & \Delta^{0} & & \Delta^{+} & & \Delta^{++} \\
s & = & -1 & & \Sigma^{*-} & & \Sigma^{*0} & & \Sigma^{*0} & \\
s & = & -2 & & & \Xi^{*-} & & \Xi^{*0} & & \\
s & = & -3 & & & & \Omega^{*-} & & & \\
 & & & I_3 = -\frac{3}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2}
\end{array}
\tag{1}
$$

with the following quark content

$$
\begin{array}{cccc}
ddd & udd & uud & uuu \\
 & sdd \quad sud \quad suu & & \\
 & ssd \quad ssu & & \\
 & sss & &
\end{array}
\tag{2}
$$

- Light octet baryons: (spin 1/2)

$$\begin{array}{lccccc} s=0 & & n & & p & \\ s=-1 & \Sigma^- & & (\Sigma^0,\Lambda) & & \Sigma^+ \\ s=-2 & & \Xi^- & & \Xi^0 & \\ I_3=-1 & -\frac{1}{2} & & 0 & \frac{1}{2} & 1 \end{array} \quad (3)$$

or in terms of quark content:

$$\begin{array}{ccc} & udd \quad uud & \\ sdd & sud & suu \\ & ssd \quad ssu & \end{array} \quad (4)$$

- Light singlet (spin 1/2): $\Lambda(uds)$

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Heavy baryons with a single heavy quark

- Heavy sextet baryons (Spin 3/2)

	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	u	d
$\Sigma_{b(c)}^{*-(0)}$	d	d
$\Xi_{b(c)}^{*0(+)}$	s	u
$\Xi_{b(c)}^{*-(0)}$	s	d
$\Omega_{b(c)}^{*-(0)}$	s	s

Table: The quark fields q_1 and q_2 for the heavy decuplet baryons.

- Heavy sextet baryons (spin 1/2)

	q_1	q_2
$\Sigma_{b(c)}^{+(++)}$	u	u
$\Sigma_{b(c)}^{0(+)}$	u	d
$\Sigma_{b(c)}^{-(0)}$	d	d
$\Xi_{b(c)}^{0(+)}$	s	u
$\Xi_{b(c)}^{-(0)}$	s	d
$\Lambda_{b(c)}^{0(+)}$	u	d

Table: The quark fields q_1 and q_2 for the heavy octet baryons.

Experimental discoveries of these baryons

- All light baryons have been discovered.
- The CDF Collaboration has observed four bottom baryons $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ [1].
- The DO [2] and CDF [3] Collaborations have seen the Ξ_b.
- The BaBar Collaboration discovered the Ω_c^* state [4].
- The CDF sensitivity appears adequate to observe new heavy baryons.

General view

- History of the QCD or SVZ sum rules.
- In this method: we see a hadron from two different windows:
  1) from the outside.
  2) we go inside it.
- The physical quantities are obtained equating these two representations and performing Double Borel transformation to suppress the contribution of higher states and continuum.

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In technique language

- we start with a correlation function where hadrons are represented by the interpolating quark currents.
- Types of the corr. func.:
  1) two point

$$T = i\int d^4x e^{ipx}\langle 0 \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0\rangle, \qquad (5)$$

  we obtain: mass, residue (lep. decay. cons.)
  2) three point

$$T = i\int d^4x d^4y e^{ipx} e^{-ipy}\langle 0 \mid T\{\eta_1(x)\eta^{tr}(0)\bar{\eta}_2(y)\} \mid 0\rangle, \qquad (6)$$

  we calculate: form factors used in decay rates, branching ratio ...

3) light cone: The main idea, here, is to expand the time ordered products of currents in the correlation function near the light cone, $x^2 \simeq 0$. Instead of the expansion of the long-distance effects in terms of operators with different mass dimensions in traditional three-point sum rules, in LCQSR , those effects are parameterized in terms of light-cone distribution amplitudes with different twists. Twist is defined as the difference between the mass dimension and the spin of local operators. In light cone sum rules, we consider T-product of two quark currents between vacuum and an on-shell state such as photon,

$$T = i \int d^4x e^{ipx} \langle \gamma \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0 \rangle, \qquad (7)$$

we obtain form factors used in electromagnetic moments, decay rates, branching ratio

This correlation function is calculated in two different approaches:

- 1) In the phenomenological side, it is saturated by a tower of hadrons with the same flavor quantum numbers.
- 2) On the quark level, it describes a hadron as quarks and gluons interacting in QCD vacuum (QCD side) via the operator product expansion (OPE), where the short- and long-distance quark-gluon interactions are separated. The former is calculated using QCD perturbation theory, whereas the latter are parameterized in terms of the vacuum condensates or light-cone distribution amplitudes.
- The physical quantities are determined matching two different representations of the correlation function.

Outline

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Phenomenological side

- Inserting the complete set of states between the interpolating currents in (46) with quantum numbers of heavy baryons.

$$T_{\mu\nu} = \frac{\langle 0 \mid \eta_\mu \mid B(p)\rangle\langle B(p) \mid \bar{\eta}_\nu \mid 0\rangle}{p^2 - m_B^2} \quad (8)$$

- The vacuum to baryon matrix element of the interpolating current is defined as

$$\langle 0 \mid \eta_\mu(0) \mid B(p,s)\rangle = \lambda_B u_\mu(p,s), \quad (9)$$

where $\lambda_B \rightarrow$ residue & $u_\mu(p,s) \rightarrow$ Rarita-Schwinger spinor.

- perform summation over spins of the spin 3/2 particles

$$\sum_s u_\mu(p,s)\bar{u}_\nu(p,s) = \frac{(\not{p}+m_)}{2m}\{-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu p_\nu}{3m^2} - \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3m}\}. \tag{10}$$

- perform summation over spins of the spin 3/2 particles

$$T_{\mu\nu} = \frac{\lambda_B^2}{p^2-m_B^2}[-\not{p}g_{\mu\nu}+...], \tag{11}$$

QCD side

- From QCD side, we need the explicit expressions of the interpolating currents in the following general form

$$\eta_\mu = A\epsilon_{abc}\left\{(q_1^{aT}C\gamma_\mu q_2^b)Q^c + (q_2^{aT}C\gamma_\mu Q^b)q_1^c + (Q^{aT}C\gamma_\mu q_1^b)q_2^c\right\}$$

where C is the charge conjugation operator and a, b and c are color indices.

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
A	$1/\sqrt{3}$	$\sqrt{2/3}$	$1/\sqrt{3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$1/\sqrt{3}$

Table: The value of A for the corresponding baryons.

- On QCD side, after contracting out the quark pairs in Eq. (46) using the Wick's theorem, we get the following expression for the correlation function in terms of quark propagators

$$\begin{aligned} & T_{\mu\nu} \\ = & -iA^2\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle 0[\gamma(q)] \mid \{S_Q^{ca'}\gamma_\nu S'^{bb'}_{q_2}\gamma_\mu S^{ac'}_{q_1} \\ + & S_Q^{cb'}\gamma_\nu S'^{aa'}_{q_1}\gamma_\mu S^{bc'}_{q_2} + S^{ca'}_{q_2}\gamma_\nu S'^{bb'}_{q_1}\gamma_\mu S^{ac'}_{Q} + S^{cb'}_{q_2}\gamma_\nu S'^{aa'}_{Q}\gamma_\mu S^{bc'}_{q_1} \\ + & S^{cb'}_{q_1}\gamma_\nu S'^{aa'}_{q_2}\gamma_\mu S^{bc'}_{Q} + S^{ca'}_{q_1}\gamma_\nu S'^{bb'}_{Q}\gamma_\mu S^{ac'}_{q_2} + Tr(\gamma_\mu S^{ab'}_{q_1}\gamma_\nu S'^{ba'}_{q_2}) \\ \times & S^{cc'}_Q + Tr(\gamma_\mu S^{ab'}_{Q}\gamma_\nu S'^{ba'}_{q_1})S^{cc'}_{q_2} + Tr(\gamma_\mu S^{ab'}_{q_2}\gamma_\nu S'^{ba'}_{Q})S^{cc'}_{q_1}\} \mid 0\rangle, \end{aligned} \tag{13}$$

where $S' = CS^TC$ and $S_Q(S_q)$ is the full heavy (light) quark propagator.

-

$$
\begin{aligned}
S_Q(x) &= S_Q^{free}(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 dv \\
&\left[\frac{\not k + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_Q^2 - k^2} v x_\mu G^{\mu\nu} \gamma_\nu \right], \\
S_q(x) &= S_q^{free}(x) - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar q q \rangle}{12} \left(1 - i \frac{m_q}{4} \not x \right) \\
&- \frac{x^2}{192} m_0^2 \langle \bar q q \rangle \left(1 - i \frac{m_q}{6} \not x \right) - ig_s \int_0^1 du \\
&\left[\frac{\not x}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma_{\mu\nu} - u x^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
&\left. - i \frac{m_q}{32\pi^2} G_{\mu\nu}\sigma^{\mu\nu} \left(\ln\left(\frac{-x^2\Lambda^2}{4} \right) + 2\gamma_E \right) \right]
\end{aligned}
\qquad (14)
$$

- the free part of the propagators are:

$$S_q^{free} = \frac{i\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2}, \tag{15}$$

$$S_Q^{free} = \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2\not{x}}{4\pi^2 x^2}K_2(m_Q\sqrt{-x^2}), \tag{16}$$

where K_i are Bessel functions.

- we obtain the following result for λ_B^2:

$$\lambda_B^2 e^{\frac{-m_{B_Q}^2}{M^2}} = A^2 \left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2) \right], \tag{17}$$

where

- $$\Pi' = \int_{m_Q^2}^{s_0} ds\, e^{-s/M^2} \Bigg\{ m_0^2 < q_1 q_1 > \left[\frac{(m_{q_1} - 6m_Q)(\psi_{22} + 2\psi_{12} - 1)}{192 m_Q^2 \pi^2} \right] - < q_1 q_1 > \left[\frac{1}{32\pi^2} [2(\psi_{02} + 2\psi_{10} - 2\psi_{21} - 1) m_Q + (\psi_{02} - 1)(3m_{q_1} - 2m_{q_2})] \right] \ldots\ldots \Bigg\}. \quad (18)$$

- The masses of the considered baryons can be determined from the sum rules. For this aim, one can get the derivative from both side of Eq. (17) with respect to $-1/M^2$ and divide the obtained result to the Eq. (17), i.e.,

$$m_{B_Q}^2 = \frac{-\frac{d}{d(1/M^2)}\left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)\right]}{\left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2)\right]}. \quad (19)$$

Numerical analysis

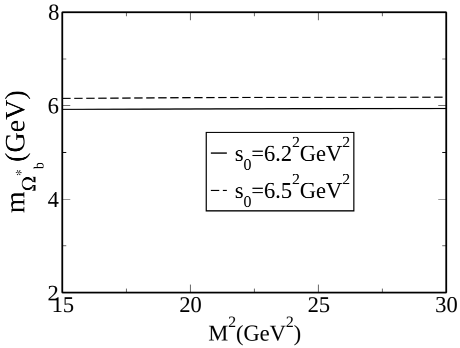

Figure: The dependence of mass of the Ω_b^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0.

	$m_{\Omega_b^*}$	$m_{\Omega_c^*}$	$m_{\Sigma_b^*}$	$m_{\Sigma_c^*}$	$m_{\Xi_b^*}$	$m_{\Xi_c^*}$
this work	6.08 ± 0.40	2.72 ± 0.20	5.85 ± 0.35	2.51 ± 0.15	5.97 ± 0.40	2.66 ± 0.18
[7]	$6.063^{+0.083}_{-0.082}$	$2.790^{+0.109}_{-0.105}$	$5.835^{+0.082}_{-0.077}$	$2.534^{+0.096}_{-0.081}$	$5.929^{+0.088}_{-0.079}$	$2.634^{+0.102}_{-0.094}$
[8]	6.088	2.768	5.834	2.518	5.963	2.654
[9]	-	-	5.805	2.495	-	-
[10]	6.090	2.770	5.850	2.520	5.980	2.650
[11]	-	2.768	-	2.518	-	-
[12]	6.083	2.760	5.840	-	5.966	-
[13]	6.060	2.752	5.871	2.5388	5.959	2.680
Exp[14]	-	2.770	5.836	2.520	-	2.645

Table: Comparison of mass of the heavy flavored baryons in *GeV* from present work and other approaches and with experiment.

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Outline

Phenomenological or physical side

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{iqx} \langle 0 \mid T\{\eta_\mu(0) J_\nu(x)\} \mid N(p)\rangle. \qquad (20)$$

$$\Pi_{\mu\nu}(p,q) = \sum_{s'} \frac{\langle 0 \mid \eta_\mu \mid \Delta^+(p',s')\rangle\langle\Delta^+(p',s') \mid J_\nu \mid N(p,s)\rangle}{m_\Delta^2 - p'^2} + \cdots, \qquad (21)$$

$$\langle 0 \mid \eta_\mu(0) \mid \Delta^+\rangle = \lambda_\Delta u^\mu(p',s'), \qquad (22)$$

$$
\begin{aligned}
& \langle \Delta(p',s') \mid J_\nu \mid N(p,s) \rangle \\
= \;& i\overline{u}^\lambda(p',s')\{(\frac{C_3^A(q^2)}{m_N}\gamma_\mu + \frac{C_4^A(q^2)}{m_N^2}p'_\mu)(g_{\lambda\nu}g_{\rho\mu} \\
- \;& g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A(q^2)g_{\lambda\nu} + \frac{C_6^A(q^2)}{m_N^2}q_\lambda q_\nu\}u(p,s),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&\Pi_{\mu\nu}(p,q) \\
= \quad & \frac{-i\lambda_\Delta}{m_\Delta^2 - p'^2}(\not{p}' + m_\Delta)\left[g_{\mu\lambda} - \frac{1}{3}\gamma_\mu\gamma_\lambda - \frac{2p'_\mu p'_\lambda}{3m_\Delta^2} + \frac{p'_\mu\gamma_\lambda - p'_\lambda\gamma_\mu}{3m_\Delta}\right]\Bigg\{ \\
& \left[\frac{C_3^A(q^2)}{m_N}\gamma_\alpha + \frac{C_4^A(q^2)}{m_N^2}p'_\alpha\right](g_{\lambda\nu}q_\alpha - q_\lambda g_{\nu\alpha}) + C_5^A(q^2)g_{\lambda\nu} \\
+ \quad & \frac{C_6^A(q^2)}{m_N^2}q_\lambda q_\nu\Bigg\} u(p). \qquad (24)
\end{aligned}
$$

We have two problems regarding the above equation: 1) all structures are not independent 2) not only spin 3/2, but spin 1/2 particles also contribute to the correlation function. To overcome these problems we choose the ordering of Dirac matrices as $\gamma_\mu \not{p}' \gamma_\nu \not{q}$. The contribution of the spin 1/2 baryons are encountered as

$$\langle 0 \mid \eta_\mu \mid \frac{1}{2}(p') \rangle = (A p'_\mu + B \gamma_\mu) u(p). \quad (25)$$

So setting the terms with γ_μ at the beginning and also terms proportional p'_μ to zero, we eliminate those contr.

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$$
\begin{aligned}
&\Pi_{\mu\nu}(p,q) \\
= &\frac{-i\lambda_\Delta}{m_\Delta^2 - p'^2}\left\{ g_{\mu\nu}\, \not{q} C_3^A(q^2) + g_{\mu\nu}\, \not{p}'\, \not{q} \frac{C_3^A(q^2)}{m_N} + g_{\mu\nu}(\not{p}' + m_\Delta) \right. \\
\times &\left(C_5^A(q^2) + C_4^A(q^2)\frac{p'q}{m_N^2} \right) - q_\mu(\not{p}' + m_\Delta)\gamma_\nu \frac{C_3^A(q^2)}{m_N} \\
- &\left. q_\mu(\not{p}' + m_\Delta)p'_\nu \frac{C_4^A(q^2)}{m_N^2} + q_\mu q_\nu(\not{p}' + m_\Delta)\frac{C_6^A(q^2)}{m_N^2} \right\} u(p) \\
+ &\text{other structures with } \gamma_\mu \text{ at the beginning or which are} \\
&\text{proportional to } p'_\mu. \qquad (26)
\end{aligned}
$$

- We will choose the structures proportional to
  $g_{\mu\nu}\ \not{p}'\ \not{q} \rightarrow C_3^A$,
  $q_\mu p'_\nu\ \not{p}' \rightarrow C_4^A$,
  $g_{\mu\nu}\ \not{p}' \rightarrow C_5^A + C_4^A \frac{p'.q}{m_\Delta^2}$,
  $q_\mu q_\nu\ \not{p}' \rightarrow C_6^A$.

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Theoretical or QCD side

- interpolating current the Δ^+

$$\eta_\mu(0) = \frac{1}{\sqrt{3}}\varepsilon^{abc}[2(u^{aT}(0)C\gamma_\mu d^b(0))u^c(0) + (u^{aT}(0)C\gamma_\mu u^b(0))d^c(0)], \quad (27)$$

- the axial current

$$J_\nu(x) = \frac{1}{2}[\overline{u}(x)\gamma_\nu\gamma_5 u(x) - \overline{d}(x)\gamma_\nu\gamma_5 d(x)]. \quad (28)$$

- $$\begin{aligned}
&\Pi_{\mu\nu}(p,q) = \\
&\frac{-1}{16\pi^2\sqrt{3}}\int\frac{d^4x e^{iqx}}{x^4}\Big\{(C\gamma_\mu)_{\alpha\beta}(\gamma_\nu\gamma_5)_{\rho\sigma} \\
&\varepsilon^{abc}\langle 0 \mid \Big[4u_\eta^a(0)u_\theta^b(x)d_\phi^c(0) \\
&\{2g_{\alpha\eta}g_{\sigma\theta}g_{\beta\phi}(\not{x})_{\lambda\rho} + 2g_{\lambda\eta}g_{\sigma\theta}g_{\beta\phi}(\not{x})_{\alpha\rho} + g_{\alpha\eta}g_{\sigma\theta}g_{\lambda\phi}(\not{x})_{\beta\rho} \\
+ \; & g_{\beta\eta}g_{\sigma\theta}g_{\lambda\phi}(\not{x})_{\alpha\rho}\} - 4u_\eta^a(0)u_\theta^b(0)d_\phi^c(x)\{2g_{\alpha\eta}g_{\lambda\theta}g_{\sigma\phi}(\not{x})_{\beta\rho} \\
+ \; & g_{\alpha\eta}g_{\beta\theta}g_{\sigma\phi}(\not{x})_{\lambda\rho}\}\Big] \mid N(p)\rangle\Big\}, \qquad (29)
\end{aligned}$$

-

$$
\begin{aligned}
& 4\langle 0|\epsilon^{abc} u_\alpha^a(a_1 x) u_\beta^b(a_2 x) d_\gamma^c(a_3 x)|P\rangle \\
= \;& \mathcal{S}_1 m_N C_{\alpha\beta}(\gamma_5 N)_\gamma + \mathcal{S}_2 m_N^2 C_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma \\
+ \;& \mathcal{P}_1 m_N (\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{P}_2 m_N^2 (\gamma_5 C)_{\alpha\beta}(\not{x} N)_\gamma \\
+ \;& (\mathcal{V}_1 + \frac{x^2 m_N^2}{4}\mathcal{V}_1^M)(\not{p} C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
+ \;& \mathcal{V}_2 m_N (\not{p} C)_{\alpha\beta}(\not{x}\gamma_5 N)_\gamma + \mathcal{V}_3 m_N (\gamma_\mu C)_{\alpha\beta}(\gamma^\mu \gamma_5 N)_\gamma \\
+ \;& \mathcal{V}_4 m_N^2 (\not{x} C)_{\alpha\beta}(\gamma_5 N)_\gamma \\
+ \;& \ldots\ldots\ldots\ldots \qquad (30)
\end{aligned}
$$

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- where, the calligraphic functions are defined in terms of the nucleon distribution amplitudes:

$$\begin{aligned} \mathcal{S}_1 &= S_1, \\ 2px\mathcal{S}_2 &= S_1 - S_2, \\ \ldots . \end{aligned} \tag{31}$$

$$\begin{aligned} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots &\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\ 4(px)^2\mathcal{V}_6 &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6, \\ \ldots\ldots\ldots\ldots \end{aligned} \tag{32}$$

-

$$
\begin{aligned}
V_1(x_i,\mu) &= 120 x_1 x_2 x_3 [\phi_3^0(\mu) + \phi_3^+(\mu)(1-3x_3)], \\
V_2(x_i,\mu) &= 24 x_1 x_2 [\phi_4^0(\mu) + \phi_3^+(\mu)(1-5x_3)], \\
V_3(x_i,\mu) &= 12 x_3 \{\psi_4^0(\mu)(1-x_3) + \psi_4^-(\mu)[x_1^2 + x_2^2 - x_3(1-x_3)] \\
&\quad + \psi_4^+(\mu)(1 - x_3 - 10 x_1 x_2)\}, \\
V_4(x_i,\mu) &= 3\{\psi_5^0(\mu)(1-x_3) + \psi_5^-(\mu)[2x_1x_2 - x_3(1-x_3)] \\
&\quad + \psi_5^+(\mu)[1 - x_3 - 2(x_1^2 + x_2^2)]\}, \\
V_5(x_i,\mu) &= 6 x_3 [\phi_5^0(\mu) + \phi_5^+(\mu)(1-2x_3)], \\
V_6(x_i,\mu) &= 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1-3x_3)], \\
A_1(x_i,\mu) &= 120 x_1 x_2 x_3 \phi_3^-(\mu)(x_2 - x_1), \\
A_2(x_i,\mu) &= 24 x_1 x_2 \phi_4^-(\mu)(x_2 - x_1),
\end{aligned}
\tag{33}
$$

....

- They contain the following functions parameterized in terms of 8 independent parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_d^1, f_d^2 and f_u^1 as

$$
\begin{aligned}
\phi_3^0 &= \phi_6^0 = f_N \\
\phi_4^0 &= \phi_5^0 = \frac{1}{2}(\lambda_1 + f_N) \\
\xi_4^0 &= \xi_5^0 = \frac{1}{6}\lambda_2 \\
\psi_4^0 &= \psi_5^0 = \frac{1}{2}(f_N - \lambda_1) \\
\phi_3^- &= \frac{21}{2}A_1^u, \\
& \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned} \tag{34}
$$

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- The numerical values are obtained using:

$$\begin{aligned} f_N &= (5.0 \pm 0.5) \times 10^{-3}\, GeV^2, \\ \lambda_1 &= -(2.7 \pm 0.9) \times 10^{-2}\, GeV^2, \\ \lambda_2 &= (5.4 \pm 1.9) \times 10^{-2}\, GeV^2. \end{aligned} \tag{35}$$

For other five independent parameters, we have used three sets as:

$$\begin{aligned} \text{Set 1 } : A_1^u &= 0.38 \pm 0.15, \quad V_1^d = 0.23 \pm 0.03, \\ f_2^d &= 0.22 \pm 0.05, \quad f_1^u = 0.07 \pm 0.05, \\ f_1^d &= 0.40 \pm 0.05, \end{aligned} \tag{36}$$

- $$\begin{aligned} \text{Set 2 } : A_1^u &= \frac{1}{14},\ \ V_1^d = \frac{13}{42},\ \ f_1^d = 0.40 \pm 0.05, \\ f_2^d &= 0.22 \pm 0.05,\ \ f_1^u = 0.07 \pm 0.05, \qquad (37) \end{aligned}$$

$$\begin{aligned} \text{Set 3 (asymptotic) } : A_1^u &= 0,\ \ V_1^d = \frac{1}{3},\ \ f_1^d = \frac{3}{10},\ \ f_2^d = \frac{4}{15}, \\ f_1^u &= \frac{1}{10} \qquad (38) \end{aligned}$$

- $$
\begin{aligned}
C_3(Q^2) &= \frac{m_N}{\sqrt{3}\lambda_\Delta} e^{\frac{m_\Delta^2}{M_B^2}} \Bigg\{ \\
& \int_{t_0}^{1} dx_2 \int_0^{1-x_2} dx_1 \frac{e^{-\frac{s(x_2,Q^2)}{M_B^2}}}{x_2} [2V_1 - T_1](x_i) \\
&+ \int_{t_0}^{1} dx_3 \int_0^{1-x_3} dx_1 \frac{e^{-\frac{s(x_3,Q^2)}{M_B^2}}}{x_3} T_1(x_i') \\
&+ \int_{t_0}^{1} dx_3 \int_0^{1-x_3} dx_1 \int_{t_0}^{x_3} \frac{dt_1}{t_1^2} e^{-\frac{s(t_1,Q^2)}{M_B^2}} \frac{m_N^2}{M_B^2} (x_3 - t_1) \\
&\times \mathcal{T}_{234578}(x_i') + \ldots\ldots\ldots\ldots
\end{aligned}
\tag{39}
$$

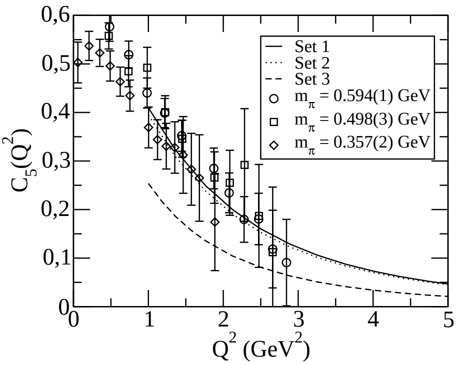

Figure: The dependence of the form factor $C_5(Q^2)$ on Q^2 for three different sets of distribution amplitudes at the continuum threshold $s_0 = 2.6\ GeV^2$ and the Borel parameter $M_B^2 = 1.5\ GeV^2$. Results from the lattice are also shown.

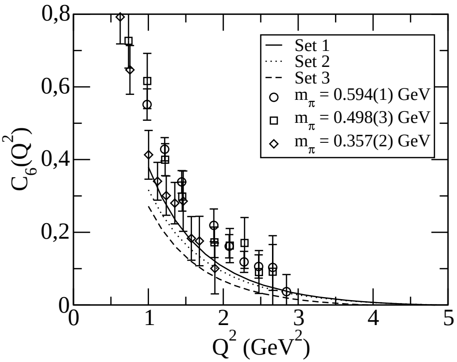

Figure: The dependence of the form factor $C_6(Q^2)$ on Q^2.

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- The electromagnetic form factors of nucleon are defined by the matrix element of the electromagnetic current J_λ^{el} between the initial and final nucleon states

$$\begin{aligned} &\langle N(p') \mid J_\lambda^{el}(0) \mid N(p)\rangle \\ = \quad &\bar{N}(p') \left[\gamma_\lambda F_1(Q^2) - \frac{i}{2m_N}\sigma_{\lambda\nu} q^\nu F_2(Q^2)\right] N(p), \end{aligned} \tag{40}$$

where $Q^2 = -q^2$, is the negative of the square of the virtual photon momentum, $q = p - p'$ and F_1 and F_2 are the Dirac and Pauli form factors, respectively.

- Another set of nucleon form factors is the so called Sachs form factors, which are defined in terms of the $F_1(Q^2)$ and $F_2(Q^2)$ as follows:

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \qquad (41)$$

At the static limit, values at $Q^2 = 0$ are $G_E^P(0) = 1$, $G_E^n(0) = 0$, $G_M^P(0) = \mu_P = 2.792847337(29)$ and $G_M^n(0) = \mu_n = -1.91304272(45)$, where μ_P and μ_n are the anomalous magnetic moments of the proton and neutron in units of the Bohr magneton.

- $$J^N(x) = 2\varepsilon^{abc}\sum_{\ell=1}^{2}(u^{Ta}(x)CA_1^\ell d^b(x))A_2^\ell u^c(x), \qquad (42)$$

  where $A_1^1 = I$, $A_1^2 = A_2^1 = \gamma_5$, $A_2^2 = \beta$, and C is the charge conjugation operator, and a, b, c are the color indices. The electromagnetic current is:

  $$J_\lambda^{el}(x) = e_u\bar{u}\gamma_\lambda u + e_d\bar{d}\gamma_\lambda d, \qquad (43)$$

  and the choice $\beta = -1$ corresponds to the Ioffe current.

- Analysis of the experimental results lead that the magnetic form factors of the nucleon are very well described by the dipole formula

$$G_M^{n,p}(Q^2) = \frac{\mu_{n,p}}{\left(1 + \frac{Q^2}{(0.71\ GeV)^2}\right)^2} = \mu_{n,p} G_D. \qquad (44)$$

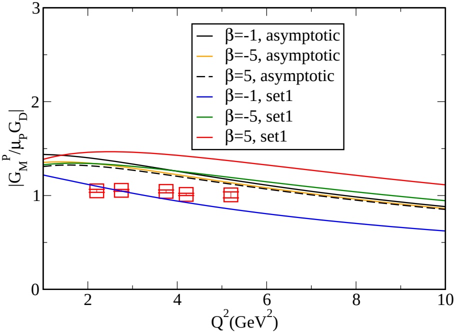

Figure: The dependence of $G_M^P/\mu_P G_D$ on Q^2 at $s_0 = 2.25\ GeV^2, M_B^2 = 1.2\ GeV^2$ for $\beta = -1,\ -5$ and 5. The boxes correspond to experimental data

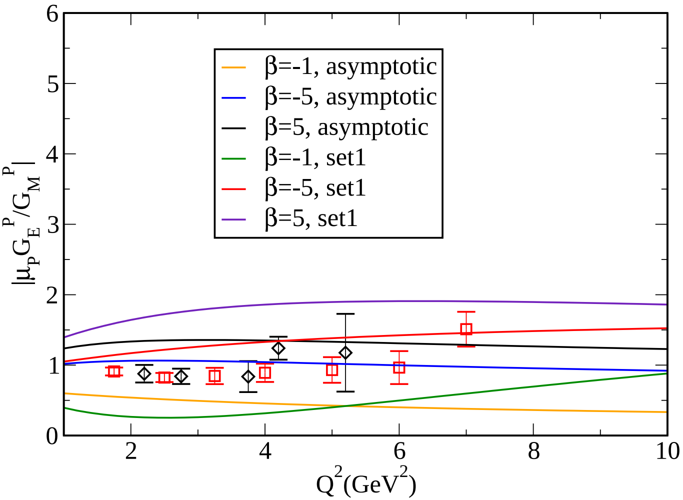
β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1
$|\mu_P G_E^P / G_M^P|$
$Q^2 (GeV^2)$
0
1
2
3
4
5
6
2
4
6
8
10

Figure:

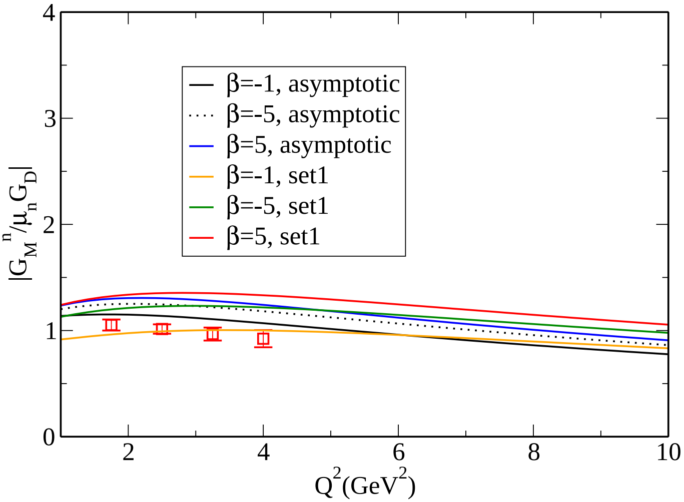
β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1
|G_M^n/μ_n G_D|
Q^2(GeV^2)
0
1
2
3
4
2
4
6
8
10

Figure:

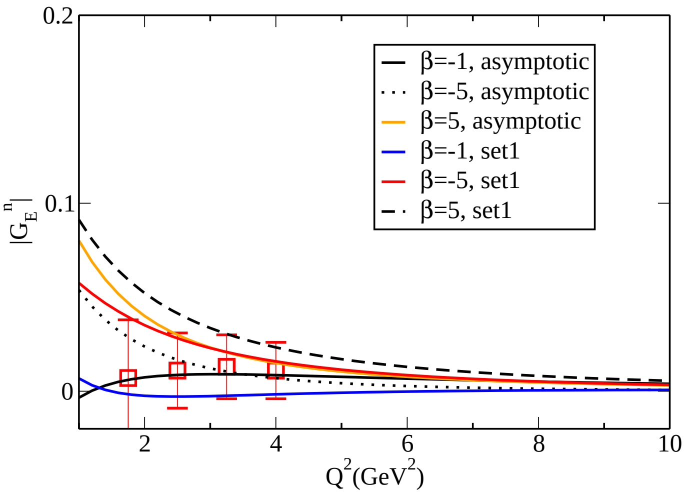
0.2
0.1
0
2
4
6
8
10
$|G_E^n|$
$Q^2(GeV^2)$
β=-1, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-5, set1
β=5, set1

Figure:

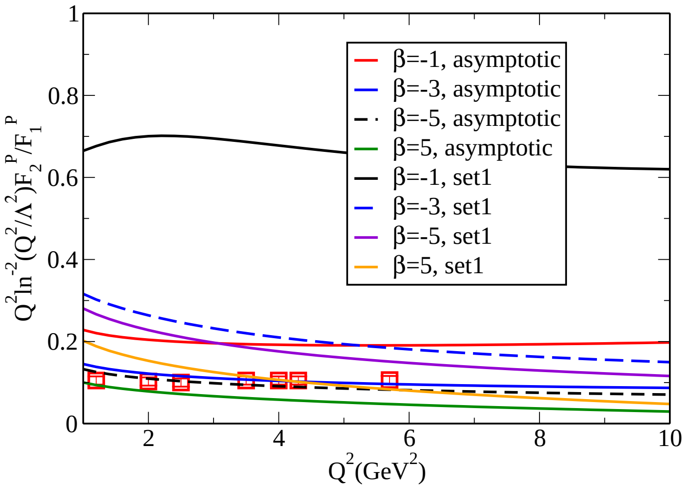
β=-1, asymptotic
β=-3, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-3, set1
β=-5, set1
β=5, set1
$Q^2 \ln^{-2}(Q^2/\Lambda^2) F_2^P / F_1^P$
$Q^2 (GeV^2)$
0
0.2
0.4
0.6
0.8
1
2
4
6
8
10

Figure:

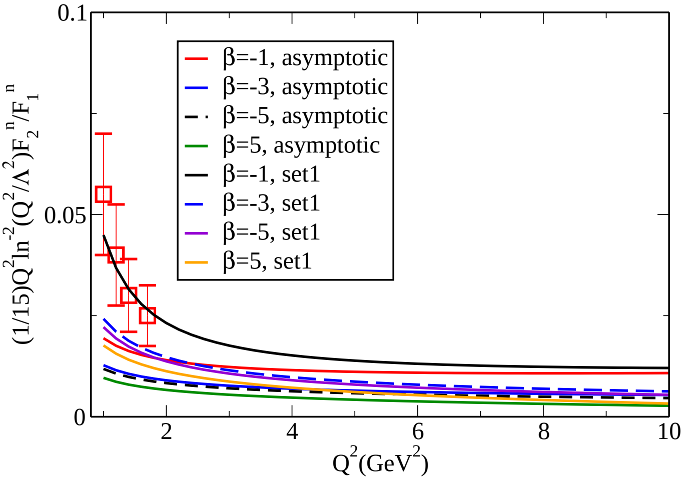
0.1
0.05
0
2
4
6
8
10
$(1/15)Q^2 \ln^{-2}(Q^2/\Lambda^2)F_2^n/F_1^n$
$Q^2(GeV^2)$
β=-1, asymptotic
β=-3, asymptotic
β=-5, asymptotic
β=5, asymptotic
β=-1, set1
β=-3, set1
β=-5, set1
β=5, set1

Figure:

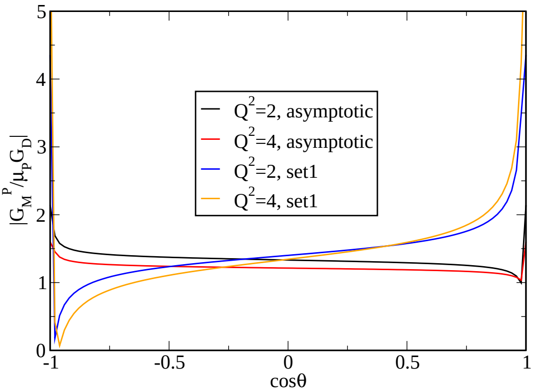

Figure: The dependence of $G_M^P/\mu_P G_D$ on $cos\theta$ at $s_0 = 2.25\ GeV^2, M_B^2 = 1.2\ GeV^2$ for two different values of Q^2, i.e. $Q^2 = 2\ GeV^2$ and $Q^2 = 4\ GeV^2$.

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- Magnetic dipole moments of the heavy spin 1/2 and 3/2 baryons

$$\Pi = i \int d^4x e^{ipx} \langle \gamma \mid T\{\eta_Q(x)\bar{\eta}_Q(0) \mid\} 0\rangle. \qquad (45)$$

$$T_{\mu\nu} = i \int d^4x e^{ipx} \langle \gamma \mid T\{\eta_\mu(x)\bar{\eta}_\nu(0)\} \mid 0\rangle, \qquad (46)$$

- $$\eta_Q = \varepsilon_{abc}[(q^{aT} C s^b)\gamma_5 + \beta(q^{aT} C\gamma_5 s^b)]Q^c, \qquad (47)$$

$$\eta_\mu = A\epsilon_{abc}\left\{(q_1^{aT} C\gamma_\mu q_2^b)Q^c + (q_2^{aT} C\gamma_\mu Q^b)q_1^c + (Q^{aT} C\gamma_\mu q_1^b)q_2^c\right\}$$

where C is the charge conjugation operator and a, b and c are color indices.

	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
A	$1/\sqrt{3}$	$\sqrt{2/3}$	$1/\sqrt{3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$1/\sqrt{3}$

Table: The value of A for the corresponding baryons.

- $$\Pi = \frac{\langle 0 \mid \eta_Q \mid \Xi_Q(p_2)\rangle}{p_2^2 - m_{\Xi_Q}^2} \langle \Xi_Q(p_2) \mid \Xi_Q(p_1)\rangle_\gamma \frac{\langle \Xi_Q(p_1) \mid \bar{\eta}_Q \mid 0\rangle}{p_1^2 - m_{\Xi_Q}^2}. \quad (49)$$

$$T_{\mu\nu} = \frac{\langle 0 \mid \eta_\mu \mid B(p_2)\rangle}{p_2^2 - m_B^2} \langle B(p_2) \mid B(p_1)\rangle_\gamma \frac{\langle B(p_1) \mid \bar{\eta}_\nu \mid 0\rangle}{p_1^2 - m_B^2}, \quad (50)$$

where $p_1 = p + q$, $p_2 = p$.

-

$$
\begin{aligned}
& < \Xi_Q(p_1) \mid \Xi_Q(p_2) >_\gamma \\
= \; & \varepsilon^\mu \overline{u}_{\Xi_Q}(p_1) \left[f_1 \gamma_\mu - i \frac{\sigma_{\mu\alpha} q_\alpha}{2m_{\Xi_Q}} f_2 \right] u_{\Xi_Q}(p_2) \\
= \; & \overline{u}_{\Xi_Q}(p_1) \left[(f_1 + f_2)\gamma_\mu + \frac{(p_1 + p_2)_\mu}{2m_{\Xi_Q}} f_2 \right] u_{\Xi_Q}(p_2) \varepsilon^\mu,
\end{aligned}
\tag{51}
$$

where, ε^μ is polarization vector of the photon.

- $$\begin{aligned}&\langle B(p_2) \mid B(p_1)\rangle_\gamma \\ =\;& \varepsilon_\rho \bar{u}_\mu(p_2)\left\{-g_{\mu\nu}\left[\gamma_\rho(f_1+f_2)+\frac{(p_1+p_2)_\rho}{2m_B}f_2+q_\rho f_3\right]\right. \\ -\;& \left.\frac{q_\mu q_\nu}{(2m_B)^2}\left[\gamma_\rho(G_1+G_2)+\frac{(p_1+p_2)_\rho}{2m_B}G_2+q_\rho G_3\right]\right\}\bar{u}_\nu(p_1), \end{aligned} \tag{52}$$

  where ε_ρ is the photon polarization vector.

- $$\langle 0 \mid \eta_Q \mid \Xi_Q(p)\rangle = \lambda_Q u_{\Xi_Q}(p), \qquad (53)$$
$$\langle 0 \mid \eta_\mu(0) \mid B(p,s)\rangle = \lambda_B u_\mu(p,s), \qquad (54)$$

- $$\Pi = -\lambda_Q^2 \varepsilon^\mu \frac{\not{p}_2 + m_{\Xi_Q}}{p_2^2 - m_{\Xi_Q}^2} \left[(f_1+f_2)\gamma_\mu + \frac{(p_1+p_2)_\mu}{2m_{\Xi_Q}} f_2\right] \frac{\not{p}_1 + m_{\Xi_Q}}{p_1^2 - m_{\Xi_Q}^2}. \quad (55)$$

  From this expression, we see that there are various structures which can be chosen for studying the magnetic moments of Ξ_Q. We choose the structure $\not{p}_2 \not{\varepsilon} \not{q}$ that contains magnetic form factor $f_1 + f_2$ and at $q^2 = 0$ it gives the magnetic moment of Ξ_Q in units of $e\hbar/2m_{\Xi_Q}$.

-

$$
\begin{aligned}
T_{\mu\nu} &= \lambda_B^2 \frac{1}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)} \Big[g_{\mu\nu}\, \not{p}\, \not{\varepsilon}\, \not{q} \frac{g_M}{3} \\
&+ \text{other structures with } \gamma_\mu \text{at the beginning and} \\
& \gamma_\nu \text{ at the end} \\
& \text{or which are proportional to } p_{2\mu} \text{or } p_{1\nu} \Big], \quad (56)
\end{aligned}
$$

where $g_M/3 = f_1 + f_2$ and at $q^2 = 0$, g_M is the magnetic moment of the baryon in units of its natural magneton, $e\hbar/2m_B c$. The factor 3 is due the fact that in the nonrelativistic limit the interaction Hamiltonian with magnetic field is equal to $g_M B = 3(f_1 + f_2)B$.

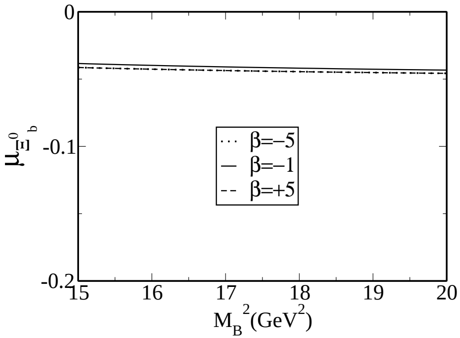

Figure: The dependence of magnetic moment $\mu_{\Xi_b^0}$ on M_B^2 at $s_0 = 6.5^2\ GeV^2$ and $\beta = \pm 5,\ -1$.

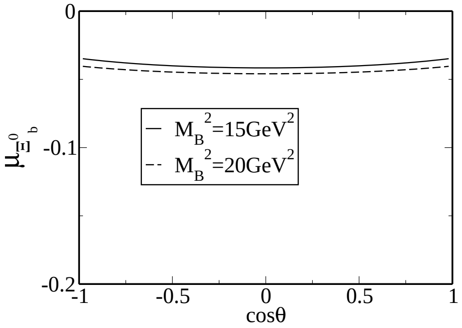

Figure: The dependence of the magnetic moment $\mu_{\Xi_b^0}$ on $cos\theta$ at $s_0 = 6.5^2\ GeV^2$ and for $M_B^2 = 15\ GeV^2$ and $M_B^2 = 20\ GeV^2$.

Table: Results for the magnetic moments of Ξ_Q baryons in different approaches.

	$\mu_{\Xi_b^0}$	$\mu_{\Xi_b^-}$	$\mu_{\Xi_c^0}$	$\mu_{\Xi_c^+}$
Our results	-0.045 ± 0.005	-0.08 ± 0.02	0.35 ± 0.05	0.50 ± 0.05
RQM [16]	-0.06	-0.06	0.39	0.41
NQM [16]	-0.06	-0.06	0.37	0.37
[5]	-	-	$-1.02 \div -1.06$	$0.45 \div 0.48$
[18]	-	-	0.32	0.42
[19]	-	-	0.38	0.38
[20]	-	-	0.28	0.28
[21]	-	-	$0.28 \div 0.34$	$0.39 \div 0.46$

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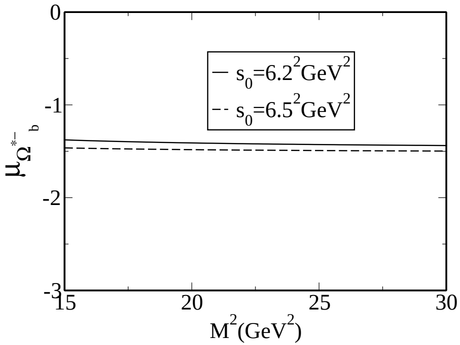

Figure: The dependence of the magnetic moment of Ω_b^{*-} on Borel parameter M^2 (in units of nucleon magneton) at two fixed values of s_0.

Table: The magnetic moments of the heavy flavored baryons in units of nucleon magneton.

	Our results	hyper central model[5]
$\mu_{\Omega_b^{*-}}$	-1.40 ± 0.35	$-1.178 \div -1.201$ $(\approx)$
$\mu_{\Omega_c^{*0}}$	-0.62 ± 0.18	$-0.827 \div -0.867$
$\mu_{\Sigma_b^{*-}}$	-1.50 ± 0.36	$-1.628 \div -1.657$
$\mu_{\Sigma_b^{*0}}$	0.50 ± 0.15	$0.778 \div 0.792$
$\mu_{\Sigma_b^{*+}}$	2.52 ± 0.50	$3.182 \div 3.239$
$\mu_{\Sigma_c^{*0}}$	-0.81 ± 0.20	$-0.826 \div -0.850$
$\mu_{\Sigma_c^{*+}}$	2.00 ± 0.46	$1.200 \div 1.256$ $(\approx)$
$\mu_{\Sigma_c^{*++}}$	4.81 ± 1.22	$3.682 \div 3.844$
$\mu_{\Xi_b^{*-}}$	-1.42 ± 0.35	$-1.048 \div -1.098$ $(\approx)$
$\mu_{\Xi_b^{*0}}$	0.50 ± 0.15	$1.024 \div 1.042$ $(\approx)$
$\mu_{\Xi_c^{*0}}$	-0.68 ± 0.18	$-0.671 \div -0.690$
$\mu_{\Xi_c^{*+}}$	1.68 ± 0.42	$1.449 \div 1.517$

Thank you again

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