Semileptonic $B_q \longrightarrow D_q^* l\nu$ $(q = s, d, u)$ Decays in QCD Sum Rules

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Outline

- $B_q(J^P = 0^-) \quad \rightarrow \quad \overline{b}q$
- $D_q^*(J^P = 1^-) \quad \rightarrow \quad \overline{c}q$
- Yields useful information for understanding the structure of the D_s^* meson.
- These decays provide possibility to calculate the elements of the CKM matrix and leptonic decay constants.
- Among the b decays, $b \rightarrow c$ transition is the most dominant transition.

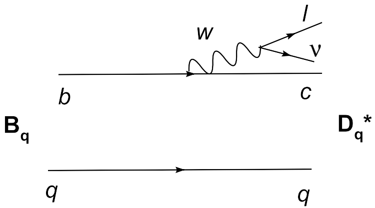
- A. A. Ovchinnikov and V. A. Slobodenyuk (1989)
  Form factors and decay widths at $q^2 = 0$ ($B \to D(D^*)$)
- V. N. Baier and A. G. Grozin (1990)
  Form factors of the $B \to D$
- P. Ball (1992)
  Calculation of the V_{cb} analysing $B \to D(D^*)e\nu$
- M. Neubert (1994)
  Form factors in HQET using the subleading Isgur-Wise form factors and $1/m_{b,c}$ corrections

Present work

- Form factors (SU(3) symmetry breaking)
- HQET limit of the form factors
- Branching ratio

$B_q \to D_q^* \ell\nu$ transitions

- The $B_q \to D_q^*$ transition proceeds by the $b \to c$ transition at the quark level



- Effective hamiltonian:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}\, \bar{l}\, \gamma_\mu (1-\gamma_5)\nu\, \bar{c}\, \gamma_\mu (1-\gamma_5) b. \qquad (1)$$

- Sandwiching the effective hamiltonian between initial and final meson states

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \, \bar{l} \, \gamma_\mu (1-\gamma_5) \nu < D_q^*(p', \varepsilon) \mid \bar{c} \, \gamma_\mu (1-\gamma_5) b \mid B_q(p) > . \qquad (2)$$

- Both vector and axial part contribute to the matrix element

- Considering Lorentz and parity invariances, this matrix element can be parameterized in terms of the form factors in the following way:

$$< D_q^*(p',\varepsilon) \mid \overline{c}\gamma_\mu b \mid B_q(p) >= i\frac{f_V(q^2)}{(m_{B_q}+m_{D_q^*})}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \quad (3)$$

$$< D_q^*(p',\varepsilon) \mid \overline{c}\gamma_\mu\gamma_5 b \mid B_q(p) > = i\left[f_0(q^2)(m_{B_q}+m_{D_q^*})\varepsilon_\mu^* - \frac{f_+(q^2)}{(m_{B_q}+m_{D_q^*})}(\varepsilon^* p)P_\mu - \frac{f_-(q^2)}{(m_{B_q}+m_{D_q^*})}(\varepsilon^* p)q_\mu\right], \quad (4)$$

where $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ are the transition form factors and $P_\mu = (p+p')_\mu$, $q_\mu = (p-p')_\mu$

QCD Sum Rules

- Shifman & Vainshtein & Zakharov (SVZ),1979
- QCD sum rules is a framework which connects hadronic parameters with QCD parameters.
- Based on the QCD Lagrangian
- The correlation function is calculated in hadrons and quark-gluon languages
- The physical quantities are determined by matching these two representations of correlators

- The correlation function

$$\Pi_{\mu\nu}^{V;A}(p^2, p'^2, q^2) = i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} < 0 \mid T[J_{\nu D_q^*}(y) J_\mu^{V;A}(0) J_{B_q}(x)] \mid 0 > \quad (5)$$

where $J_{\nu D_q^*}(y) = \overline{q}\gamma_\nu c$ and $J_{B_q}(x) = \overline{b}\gamma_5 q$ are the interpolating currents of D_q^* and B_q mesons, respectively and $J_\mu^V = \overline{c}\gamma_\mu b$ and $J_\mu^A = \overline{c}\gamma_\mu\gamma_5 b$

Outline

- Phenomenological part of the correlation function

$$\Pi_{\mu\nu}^{V,A}(p^2, p'^2, q^2) = \frac{< 0 \mid J^{\nu}_{D_q^*} \mid D_q^*(p', \varepsilon) >< D_q^*(p', \varepsilon) \mid J_{\mu}^{V,A} \mid B_q(p) >< B_q(p) \mid J_{B_q} \mid 0 >}{(p'^2 - m_{D_q^*}^2)(p^2 - m_{B_q}^2)} + \cdots \quad (6)$$

... contributions coming from higher states and continuum

- By definitions:

$$< 0 \mid J^{\nu}_{D_q^*} \mid D_q^*(p', \varepsilon) >= f_{D_q^*} m_{D_q^*} \varepsilon^{\nu} \;, \;\; < B_q(p) \mid J_{B_q} \mid 0 >= -i \frac{f_{B_q} m_{B_q}^2}{m_b + m_q} , \quad (7)$$

- Performing summation over the polarization of the D_q^* meson

$$
\begin{aligned}
\Pi^A_{\mu\nu}(p^2, p'^2, q^2) &= \frac{f_{B_q} m_{B_q}^2}{(m_b + m_q)} \frac{f_{D_q^*} m_{D_q^*}}{(p'^2 - m_{D_q^*}^2)(p^2 - m_{B_q}^2)} \\
&\times [-f_0 g_{\mu\nu}(m_{B_q} + m_{D_q^*}) + \frac{f_+ P_\mu p_\nu}{(m_{B_q} + m_{D_q^*})} + \frac{f_- q_\mu p_\nu}{(m_{B_q} + m_{D_q^*})}] \\
&+ \text{excited states}, \\
\Pi^V_{\mu\nu}(p^2, p'^2, q^2) &= -\varepsilon_{\alpha\beta\mu\nu} p^\alpha p'^\beta \frac{f_{B_q} m_{B_q}^2}{(m_b + m_s)(m_{B_q} + m_{D_q^*})} \frac{f_{D_q^*} m_{D_q^*}}{(p'^2 - m_{D_q^*}^2)(p^2 - m_{B_q}^2)} f_V \\
&+ \text{excited states}. \qquad (8)
\end{aligned}
$$

- We chose $i\varepsilon_{\mu\nu\alpha\beta} p'^\alpha p^\beta$, $g_{\mu\nu}$ and $\frac{1}{2}(p_\mu p_\nu \pm p'_\mu p_\nu)$ for f_V, f_0 and $f_\pm$, respectively.

Outline

Theoretical Part

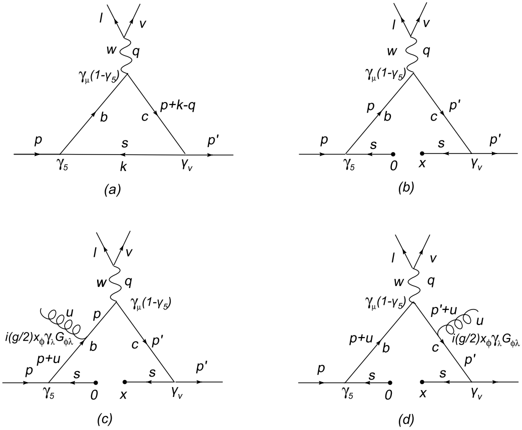


Figure: Feynman diagrams for $B_q \to D_q^* l \nu$ $(q = s, d, u)$ transitions.

- The theoretical part of the correlation function is calculated by means of OPE, and up to operators having dimension $d = 5$, it is determined by the bare-loop (Fig. 1 a) and the power corrections (Fig. 1 b, c, d) from the operators with $d = 3$, $< \overline{\psi}\psi >$, $d = 4$, $m_s < \overline{\psi}\psi >$, $d = 5$, $m_0^2 < \overline{\psi}\psi >$
- The QCD sum rules for the f_V, f_0, f_+ and f_- is obtained by equating the phenomnological expression and the OPE expression and applying double Borel transformations with respect to the variables p^2 and p'^2

$$f_i(q^2) = \kappa \frac{(m_b + m_q)}{f_{B_q} m_{B_q}^2} \frac{\eta}{f_{D_q^*} m_{D_q^*}} e^{m_{B_q}^2/M_1^2 + m_{D_q^*}^2/M_2^2}$$
$$\times [\frac{1}{(2\pi)^2} \int_{(m_c+m_s)^2}^{s_0'} ds' \int_{f(s')}^{s_0} ds \rho_i(s,s',q^2) e^{-s/M_1^2 - s'/M_2^2}$$
$$+ \hat{B}(f_i^{(3)} + f_i^{(4)} + f_i^{(5)})] \qquad (9)$$

where $\eta = m_{B_q} + m_{D_q^*}$ for $i = V, \pm$ and $\eta = \frac{1}{m_{B_q} + m_{D_q^*}}$ for $i = 0$ are considered. Here $\kappa = +1$ for $i = \pm$ and $\kappa = -1$ for $i = 0$ and V

- To subtract the contributions of the higher states and the continuum, the quark-hadron duality assumption is used.

- The double Borel transformations

$$\hat{B}\frac{1}{r^m}\frac{1}{r'^n} \to (-1)^{m+n}\frac{1}{\Gamma(m)}\frac{1}{\Gamma(n)}e^{-m_b^2/M_1^2}e^{-m_c^2/M_2^2}\frac{1}{(M_1^2)^{m-1}(M_2^2)^{n-1}}. \tag{10}$$

- The $f(s')$ is determined

$$-1 \leq \frac{2ss' + (s+s'-q^2)(m_b^2 - s - m_q^2) + (m_q^2 - m_c^2)2s}{\lambda^{1/2}(m_b^2, s, m_q^2)\lambda^{1/2}(s, s', q^2)} \leq +1. \tag{11}$$

- The usual triangle function

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab \tag{12}$$

Outline

Form Factors

- The allowed physical region for q^2 is $m_l^2 \leq q^2 \leq (m_{B_q} - m_{D_q^*})^2$

$f_i(0)$	$B_s \to D_s^* \ell\nu$	$B_d \to D_d^* \ell\nu$	$B_u \to D_u^* \ell\nu$
$f_V(0)$	0.36 ± 0.08	0.47 ± 0.13	0.46 ± 0.13
$f_0(0)$	0.17 ± 0.03	0.24 ± 0.05	0.24 ± 0.05
$f_+(0)$	0.11 ± 0.02	0.14 ± 0.025	0.13 ± 0.025
$f_-(0)$	-0.13 ± 0.03	-0.16 ± 0.04	-0.15 ± 0.04

Table: The value of the form factors at $q^2 = 0$

- Our form factors work in the inteval $0 \leq q^2 \leq 10 GeV^2$ and truncated at $\simeq 10\ GeV^2$

- To extend of the results to whole physical region we look for a fit parameterizations

$$f_i(q^2) = \frac{f_i(0)}{1 + \alpha\hat{q} + \beta\hat{q}^2 + \gamma\hat{q}^3 + \lambda\hat{q}^4}, \qquad (13)$$

where $\hat{q} = q^2/m_{B_q}^2$.

	f(0)	α	β	γ	λ
f_V	0.38	-2.53	2.77	-2.41	0.03
f_0	0.18	-1.77	0.98	-0.23	-3.50
f_+	0.12	-2.90	3.66	-3.72	-1.69
f_-	-0.15	-2.63	2.72	-0.99	-6.48

Table: Parameters appearing in the form factors of the $B_s \to D_s^*(2112)\ell\nu$ for $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$.

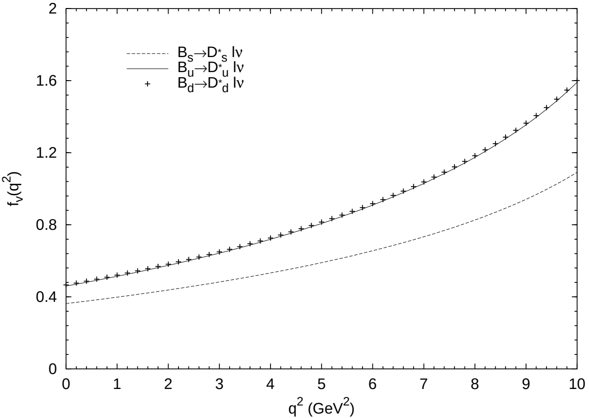


Figure: The dependence of f_V on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

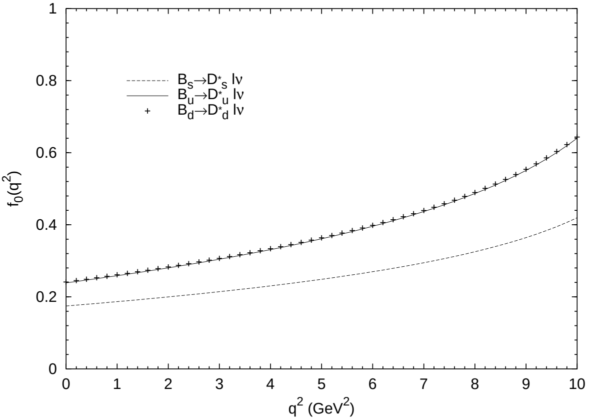


Figure: The dependence of f_0 on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

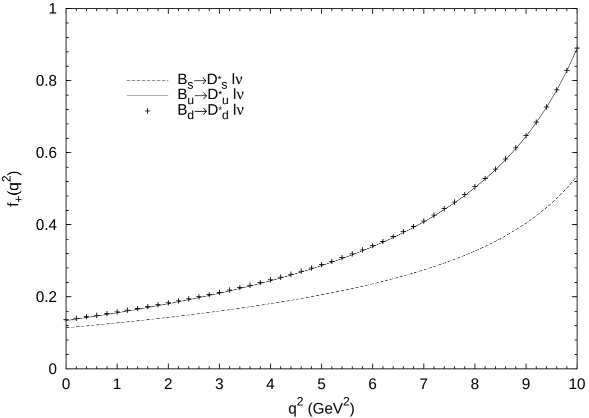


Figure: The dependence of f_+ on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

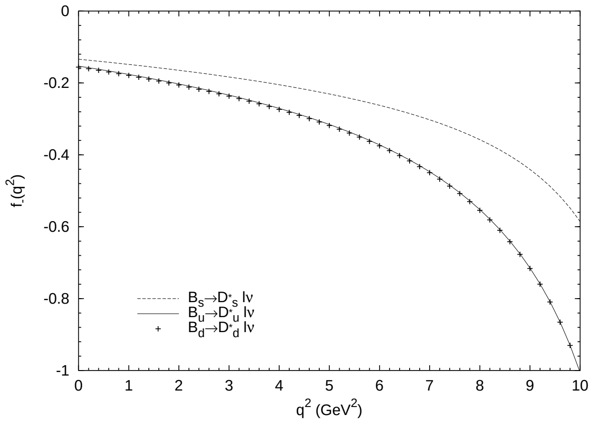


Figure: The dependence of f_- on q^2 at $M_1^2 = 19\ GeV^2$, $M_2^2 = 5\ GeV^2$, $s_0 = 35\ GeV^2$ and $s_0' = 6\ GeV^2$.

Outline

HQET

- $m_b \to \infty$, $m_c = \frac{m_b}{\sqrt{z}}$, $\sqrt{z} = y + \sqrt{y^2 - 1}$

$$y = \nu\nu' = \frac{m_{B_q}^2 + m_{D_q^*}^2 - q^2}{2 m_{B_q} m_{D_q^*}} \quad (14)$$

  where ν and ν' are the four-velocities of the initial and final meson states
- $M_1^2 = 2T_1 m_b$ and $M_2^2 = 2T_2 m_c$

$$\begin{aligned} R_{1(2)[3]} &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_{V(+)[-]}(y)}{f_0(y)}, \\ R_{4(5)} &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_{+(-)}(y)}{f_V(y)}, \\ R_6 &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{f_-(y)}{f_+(y)}. \end{aligned} \quad (15)$$

y	1 (zero recoil)	1.1	1.2	1.3	1.4	1.5
$q^2(GeV^2)$	10.69	8.57	6.45	4.33	2.20	0.08
R_1	1.34	1.31	1.25	1.19	1.10	0.95
R_2	0.80	0.99	1.10	1.22	1.30	1.41
R_3	-0.80	-0.79	-0.80	-0.81	-0.80	-0.80
R_4	0.50	0.64	0.77	0.94	1.20	1.46
R_5	-0.50	-0.51	-0.56	-0.62	-0.71	-0.89
R_6	-0.80	-0.67	-0.64	-0.61	-0.55	-0.53
R_1 [1]	1.31	1.30	1.29	1.28	1.27	1.26
R_2 [1]	0.90	0.90	0.91	0.92	0.92	0.93

Table: The values for the R_i and comparison of $R_{1,2}$ values with the predictions of [1].

Outline

	$B_s \to D_s^* \ell\nu$	$B_d \to D_d^* \ell\nu$	$B_u \to D_u^* \ell\nu$
Present study	$(1.89 - 6.61) \times 10^{-2}$	$(4.36 - 8.94) \times 10^{-2}$	$(4.57 - 9.12) \times 10^{-2}$
CQM model	$(7.49 - 7.66) \times 10^{-2}$	$(5.9 - 7.6) \times 10^{-2}$	$(5.9 - 7.6) \times 10^{-2}$
Experiment	-	$(5.35 \pm 0.20) \times 10^{-2}$	$(6.5 \pm 0.5) \times 10^{-2}$

Table: Comparison of the branching ratio of the $B_q \to D_q^* \ell\nu$ decays in present study, the CQM model [2] and the experiment [3].

- M. Neubert, Phys. Rev. **D46** (1992) 3914.
- Shu-Min Zhao, Xiang Liu, Shuang-Jiu Li, Eur. Phy. J. **C51** (2007) 601.
- W.M. Yao et al., Particle Data Group, J. Phys. **G33** (2006) 1.