COTTON VE RICCI AKISI

*More on Cotton Flow on Three Manifolds (E. Kılıçarslan, S. Dengiz, B.T.), 2015

*Cotton Flow (Ö. Kişisel, Ö Sarıoğlu, B.T.), 2008

13.02.2015
YEF, Ankara

Heat/Diffusion Equation

$$\frac{\partial U}{\partial t} = \nabla^2 U$$

Heat Equation

- Temperature of the HOTTEST point is non-increasing.

- Temperature of the COLDEST point is non-decreasing.

- So wait long enough temperature will be uniform. (Why ? Think About it !)

The Earth

Close up look

DEVIATIONS FROM Smoothness

- $e = \frac{h}{R} = \frac{1}{10^3}$ for the Earth
- $e = \frac{1}{10^6}$ for the Neutron Star

(So the tallest mountain on a Neutron star is around 1 cm !)

- $e = 0$ for a Black Hole

Surface Gravity

- So, can we write a diffusion type equation for gravity, something like

$$\frac{\partial U}{\partial t} = \nabla^2 U$$

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- with $U = g \sim 9.8\ m/s^2$ for the Earth.
- We can, but after Einstein we learned that the theory of gravity is General Relativity and instead of the acceleration g we should use the metric.

METRIC

$$g = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix}$$

So;

Perhaps we can write an equation of the form with **$U = g_{ij}$**

$$\frac{\partial U}{\partial t} = \nabla^2 U$$

- BUT, this equation does <u>not</u> make much sense for several reasons

   1. Coordinate dependent

   2. Not consistent with relativity

Einsten’s Equation

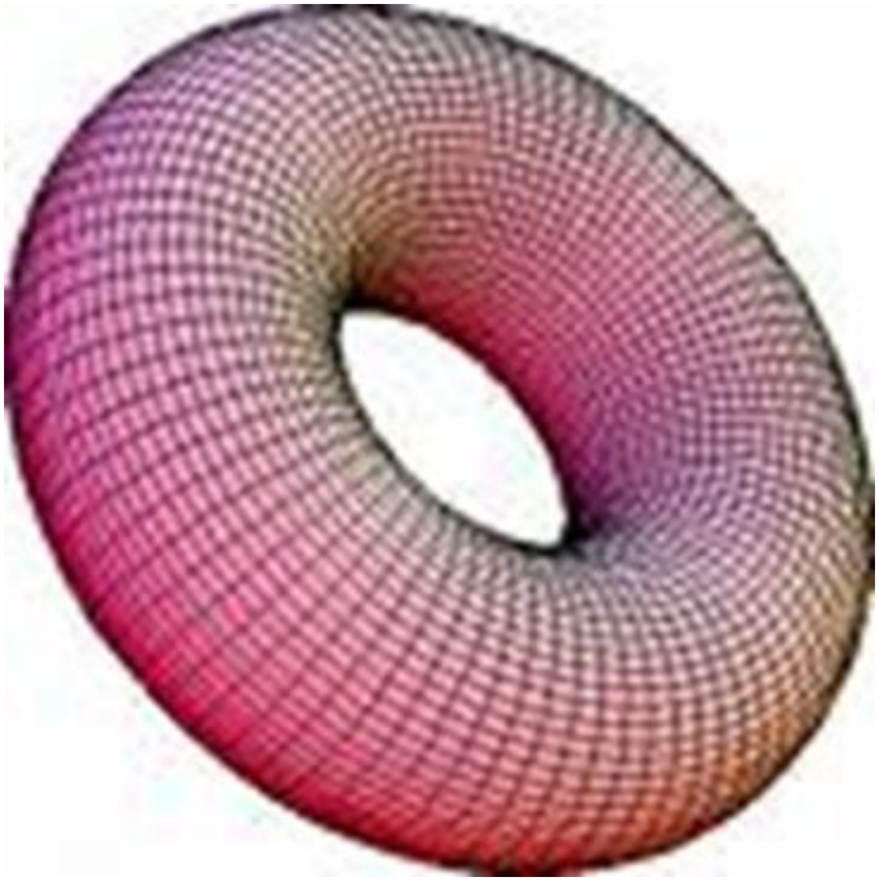
$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8 \pi G}{c^4} T_{\mu\nu}$$

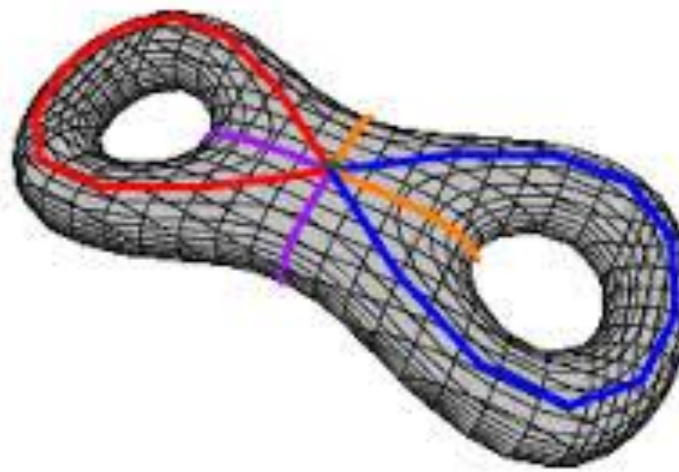
- So this equation must have a smoothing behavior.

- Yes it does, but it also has black holes, singularities etc....

Enter Mathematics

Poincare Conjecture






Wilson
Wilson

1904

- Poincare’ conjectured that in dimension there is an analogy of his sphere theorem.


22 December 2006 | $10
Science
Breakthrough
of the Year
The
Poincaré
Conjecture
PROVED
AAAS


Grigory Perelman 1966-x

Ricci Flow

$$\frac{\partial}{\partial t} g_{ij} = -2R_{ij}.$$

Richard Hamilton

Diffusive equation

$$\frac{\partial}{\partial t} g_{ij} = \Delta(g_{ij}) + Q_{ij}(g^{-1}, \partial g).$$

What Perelman did ?

- Proved 3 dimensional version of uniformization theorem (so called Thurston’s conjecture)

Cotton Flow

Cotton Flow Equation

$$\frac{\partial}{\partial t} g_{ij}(t,x) = \kappa C_{ij}(t,x)$$

RESULTS

- Conformally flat metrics are critical points
- Nicest Conformally flat metrics, Einstein metrics are not stable