Study of $\chi_{c0}(1P) \to J/\psi\gamma$ and $\chi_{b0}(1P) \to \Upsilon(1S)\gamma$ decays via QCD sum rules

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Outline

- Heavy quarkonium states(like $b\bar{b}$ and $c\bar{c}$) and their decay modes offer a laboratory to study the strong interaction in the non-perturbative regime. Charmonium in particular has served as a calibration tool for the corresponding techniques and models [1].
- Heavy quarkonium states can have many bound states and decay channels used to study and determine different parameters of Standard Model(SM) and QCD from the theoretical perspective.

- In particular, the calculation of bottomonium masses[2], total widths, coupling constants[6, 5, 3, 4] and branching ratio can serve as benchmarks for the low energy predictions of QCD.
- We present the theoretical study on the form factor of exclusive $\chi_{c0} \to J/\psi\gamma$ and $\chi_{b0} \to \Upsilon\gamma$ decays in the frame work of QCD sum rules. Note that in order to calculate the branching ratio we have to acquire information about the masses and decay constants of the participating particles.
- The masses can be obtained either by means of the experimental results i.e, the Particle Data Group or by the theoretical methods . The decay constants, on the other hand, can be calculated theoretically via different non-perturbative methods.

QCD Sum Rules for the form factors

The method: QCD Sum Rules
The starting point: Three-point correlation function

$$\Pi_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{-ip\cdot x + ip'\cdot y} \langle 0 | \mathcal{T}\left(j_\mu^V(y) j_\nu^{em}(x) \bar{j}^S(0) \right) | 0 \rangle \tag{1}$$

where $\mathcal{T}$ is the time ordering operator and q is momentum of photon. Each meson and photon field can be described in terms of the quark field operators as follows:

$$\begin{aligned} j_\mu^V(y) &= \overline{c(b)}(y)\gamma_\mu c(b)(y) \\ j^S(x) &= \overline{c(b)}(x) c(b)(x) \\ j_\mu^{em}(x) &= Q_{c(b)}\overline{c(b)}(x)\gamma_\mu c(b)(x) \end{aligned} \tag{2}$$

The correlation function Eq. (1) in two different methods. In phenomenological or physical approach, it can be evaluated in terms of hadronic parameters such as masses, decay constants and form factors. In theoretical or QCD side, on the other hand, it is calculated in terms of QCD parameters, which are quark and gluon degrees of freedom, by the help of the operator product expansion (OPE) in deep Euclidean region.Equating the structure calculated in two different approaches of the same correlation function, we get a relation between hadronic parameters and QCD degrees of freedom. Finally, we apply double Borel transformation with respect to the momentum of initial and final mesons(p^2 and p'^2). This final operation suppresses the contribution of the higher states and continuum.

Phenomenological side

Correlation function after inserting the complete set of the mesons and integrating over x and y is:

$$\Pi_{\mu\nu} = \frac{\langle 0|j^S(x)|S\rangle\langle S|j_\nu^{em}(x)|V\rangle\langle V|\bar{j}_\mu^V|0\rangle}{(m_V^2 - p'^2)(m_S^2 - p^2)} + \ldots \tag{3}$$

where .... contains the contribution of the higher and continuum states with the same quantum numbers . The matrix elements of the above equation are related to the hadronic parameters as follows:

$$\begin{aligned} \langle 0|j_\mu^V(x)|V\rangle &= m_V f_V \epsilon'_\mu \\ \langle S|\bar{j}^S|0\rangle &= i m_S f_S^* \\ \langle S|j_\nu^{em}(x)|V\rangle &= eF(q^2=0)\{(p'\cdot q)\epsilon'_\nu - (q\cdot\epsilon')p'_\nu\} \end{aligned} \tag{4}$$

where $F(q^2)$ is the form factor of transition and ϵ' is the polarization vector associated with the vector meson.

Using Eq. (4) in Eq. (3) and considering the summation over polarization vectors via,

$$\epsilon_\nu \epsilon_\theta^* = -g_{\nu\theta},$$
$$\epsilon'_j \epsilon'^*_\mu = -g_{j\mu} + \frac{p'_j p'_\mu}{m_V^2}, \qquad (5)$$

the result of the physical side is as follows:

$$\Pi_{\mu\nu} = -\frac{e m_V f_V m_S f_S^*}{(m_V^2 - p'^2)(m_S^2 - p^2)} F(0)(p' \cdot q) g_{\mu\nu} + ... \qquad (6)$$

We are going to compare the coefficient of $g_{\mu\nu}$ structure for further calculation from different approaches of the correlation functions.

Theoretical(QCD) side

The QCD side of the calculation is made in deep Euclidean region where $p^2 \to -\infty$ and $p'^2 \to -\infty$. Theoretical side consists of perturbative(bare loop see fig. (1) and non-perturbative parts(the contributions of two gluon condensate diagrams fig. (2)).

$$\Pi_{\mu\nu}(p', p) \quad = \quad (\Pi_{per} + \Pi_{nonper})\,(p' \cdot q) g_{\mu\nu}, \qquad (7)$$

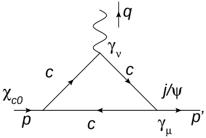
bare loop

The perturbative part is a double dispersion integral as follows:

$$\Pi_{per} = -\frac{1}{4\pi^2}\int ds' \int ds \frac{\rho(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}, \quad (8)$$

where, $\rho(s,s',q^2)$ is called spectral density. The generic methods to calculate this bare loop integral is the Cutkosky method, where the quark propagators of Feynman integrals are replaced by the Dirac delta functions. Then, using the Cutkosky method we get spectral density as:

$$\rho(s,s',q^2) = \frac{2m_{c(b)}N_c(-4m_{c(b)}^2 + q^2 + s - s')}{3\lambda^{1/2}(s,s',q^2)(q^2+s-s')}, \quad (9)$$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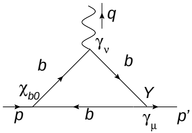
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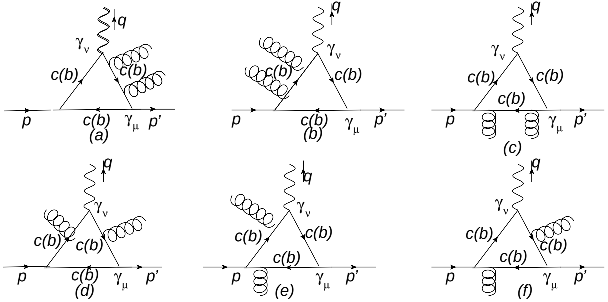
Three Delta functions vanish simultaneously, then:

$$-1 \leq f(s,s') = \frac{s(q^2+s-s')}{\lambda^{1/2}(m^2_{c(b)}, m^2_{c(b)}, s)\lambda^{1/2}(s,s',q^2)} \leq 1, \quad (10)$$

Two Gluon Condensates

Fig. (2) shows the diagrams considered for gluon condensates. Note that heavy quark condensates vanish.



Our results for theses diagrams after Borel transformations are as:

$$\begin{aligned}
\Pi_{nonper} &= -\frac{2\pi\alpha_s\langle G\rangle^2}{3}16m_{c(b)})(-\hat{I}_0(1,2,2)+6\hat{I}_0(1,3,1)+\hat{I}_0(2,1,2)\\
&- 2(\hat{I}_0(2,2,1)-2\hat{I}_1(1,2,2)-6\hat{I}_1(1,3,1)+2\hat{I}_1(2,1,2)\\
&- 6\hat{I}_1(2,2,1)+\hat{I}_1(3,1,1)+3m_{c(b)})2(\hat{I}_0(1,1,4)+2(\hat{I}_0(1,4,1)\\
&+ \hat{I}_0(4,1,1)+\hat{I}_1(1,1,4)+2(\hat{I}_1(1,4,1)+\hat{I}_1(4,1,1))))\\
&- 3\hat{I}_2(1,1,3))+6\hat{I}_2(1,3,1)) \qquad (11)
\end{aligned}$$

Now, we can compare $g_{\mu\nu}$ coefficient of Eq.(6) and Eq. (7) . Our result related to the sum rules for the corresponding form factor is as follows:

$$F(q^2) = \frac{e^{\frac{m_S^2}{M^2}} e^{\frac{m_V^2}{M'^2}}}{f_V f_S m_V m_S} \left[\frac{1}{4\,\pi^2} \int_{4m_{c(b)}^2}^{s_0} ds \int_{4m_{c(b)}^2}^{s_0'} ds' \rho(s,s',q^2) \theta[1-(f(s,s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} + \Pi_{nonper} \right], \quad (12)$$

Note that, finally we have to set $q^2 = 0$ for the real photon.

In this section we calculate the value of form factors and the branching ratios. We use, $m_c = 1.275 \pm 0.025\ GeV$, $m_b = 4.65 \pm 0.03\ GeV$[30], $m_{J/\psi} = 3096.916 \pm 0.011 MeV$[30], $m_{\chi_{c0}} = 3414.75 \pm 0.31 MeV$[30],$m_{\chi_{b0}} = 9859.44 \pm 0.42 \pm 0.31\ MeV$[30],$m_{\Upsilon} = 9460.30 \pm 0.26$MeV[30], $f_{\chi_{c0}} = (343 \pm 112)\ MeV$[31], $f_{\chi_{b0}} = (175 \pm 55)\ MeV$[31], $f_{J/\psi} = (481 \pm 36)\ MeV$[2], $f_{\Upsilon} = (746 \pm 62)\ MeV$ [2] and the full width for χ_c: $\Gamma_{tot}^{\chi_c} = 10.4 \pm 0.6$ MeV[30].

the continuum thresholds(s_0 and s_0') and the Borel mass parameters(M^2 and M'^2). The physical results are required to be either weakly depend on or independent of aforementioned parameters. Therefore, we must consider the working regions of these auxiliary parameters where the dependence of the form factors are weak. We also consider the working regions for the Borel mass parameters M^2 and M'^2 in a way that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensions operators can be ignored.

- We find the stable region for the form factor in the following intervals: $12\ GeV^2 \leq M^2 \leq 25\ GeV^2$ and $10\ GeV^2 \leq M'^2 \leq 20\ GeV^2$ for $\chi_{c0} \to J/\psi\gamma$ decays(see also Fig. (**??**) and (**??**)). We also get $15\ GeV^2 \leq M^2 \leq 30\ GeV^2$ and $15\ GeV^2 \leq M'^2 \leq 30\ GeV^2$ for $\chi_{b0} \to \Upsilon\gamma$ decays.
- The continuum thresholds, s_0 and s'_0 are fixed by the mass of the corresponding ground-state hadron. Note that they must not be greater than the energy of the first excited states with the same quantum numbers. In our numerical calculations the following regions for the continuum thresholds in s and s' channels are used: $(m_S + 0.3)^2 \leq s_0 \leq (m_S + 0.7)^2$ and $(m_V + 0.3)^2 \leq s'_0 \leq (m_V + 0.7)^2$ for s and s' channels, respectively.
- Note that, we follow the standard procedure in the QCD sum rules, where the continuum thresholds are supposed

The best fit curve for Eq.(12) in the negative q^2 region is employed as:

$$F(q^2) = ae^{-bq^2} + c \tag{13}$$

where we $a = 0.73 \pm 0.26$, $b = -0.2 \pm 0.01$ and $c = 0.012 \pm 0.02$ for $\chi_{c0} \to J/\psi\gamma$ and $a = 0.4812 \pm 0.18$, $b = 0.2 \pm 0.01$ and $c = 0.0084 \pm 0.003$ for $\chi_{b0} \to \Upsilon\gamma$ decays. This fit is extrapolated for positive q^2 region (see Fig. (3)). In other words, we use the analytical continuation of the form factor from negative q^2 into the physical region. (This is based on the principle of the QCD sum rules method.):
Using $q^2 = 0$ in Eq. (13), we obtain the $F(0) = 0.73 \pm 0.27\ GeV^{-1}$ and the $F(0) = 0.47 \pm 0.14\ GeV^{-1}$ for $\chi_{c0} \to J/\psi\gamma$ and $\chi_{b0} \to \Upsilon\gamma$ decays, respectively.
The errors in our numerical calculation are the results of both the interval of the working regions for the auxiliary parameters

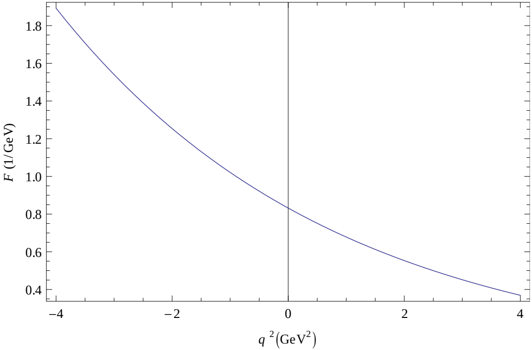


Figure: The dependence of the fit function (Eq. (13)) and the form factor(Eq.(12)) on q^2 for values of $s_0 = 14.5$, $s_0' = 12.25$, $M^2 = 15$ and $M'^2 = 12$ for the $\chi_{c0} \to J/\psi\gamma$ decays.

The matrix element for the decay of $\chi_{c0} \to J/\psi\gamma$ and $\chi_{b0} \to \Upsilon\gamma$ is as follows:

$$M = eF(q^2 = 0)\{(p' \cdot q)\epsilon' \cdot \epsilon - (q \cdot \epsilon')(p' \cdot \epsilon)\} \quad (14)$$

where p' and ϵ' are the momentum and polarization of final state vector meson i.e., either J/ψ or Υ mesons, ϵ is the polarization of real photon and p is the momentum of initial scalar meson.
Using this matrix element, we get:

$$\Gamma = \frac{|\overrightarrow{p}|}{8\pi m_S^2}|M|^2 = \frac{\alpha}{8}F^2(0)m_S^3(1 - \frac{m_V^2}{m_S^2})^3 \quad (15)$$

where m_S is mass of either χ_{c0} or χ_{b0} meson and m_V is either mass of J/ψ or Υ meson. The decay width for $\chi_{c0} \to J/\psi\gamma$ decays is as:

$$\Gamma(\chi_{c0} \to J/\psi\gamma) = (11.2 \pm 3.3) \times 10^{-5}\text{GeV}. \quad (16)$$

The branching ratio of $\chi_{c0} \to J/\psi\gamma$ can be evaluated with the Eq. (16)and using the experimental total width that is as:

$$\mathcal{B}_r(\chi_{c0} \to J/\psi\gamma) = (1.07 \pm 0.34) \times 10^{-2} \tag{17}$$

this result is in good agreement with the experimental measurement[30] which is:

$$\mathcal{B}_r(\chi_{c0} \to J/\psi\gamma) = (1.17 \times \pm 0.08) \times 10^{-2}. \tag{18}$$

We get $F(0) = 0.47 \pm 0.13\ GeV^{-1}$ for $\chi_{b0} \to \Upsilon\gamma$ decays. Using this value we calculate the decay width as follows:

$$\Gamma(\chi_{b0} \to \Upsilon\gamma) = (9.9 \pm 2.8) \times 10^{-5}\text{GeV}. \tag{19}$$

This decay width and the measured branching ratio $\mathcal{B}_r(\chi_b \to \Upsilon\gamma) = (1.76 \pm 0.30) \times 10^{-2}$[15] allow us to evaluate the total width of χ_{b0}. We estimate that the Full Width $\Gamma_{tot}(\chi_{b0}(1P)) = 5.5 \pm 1.5\ MeV$, which is consistent with the experimental results that indicate the Full Width $\Gamma_{tot} < 6\ MeV$[15]

To sum up, in this work we calculate the form factors for the exclusive $\chi_{c0} \to J/\psi\gamma$ and $\chi_{b0} \to \Upsilon\gamma$ decays using the QCD sum rules method. The results of the form factors are used to determine the decay widths and the branching ratios of the aforementioned decays. transition.

THANK YOU...

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