CPT Violating QED and Beyond

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Outline

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What is the aim?

The problem of marrying Quantum Field Theory with General Relativity is unsolved as ever!

The direct experimental measurements of unifying theory in Planck Scale is unreachable for foreseeable future!

Instead, one can search for Quantum Gravity effects in low energy $\Rightarrow$ Look for exotic phenomena that can NOT occur either in QFT or in GR!

Why not Lorentz & CPT violation?

- Lorentz Symmetry is conserved both in QFT and GR: *A good candidate to check!*
- Most Quantum Gravity candidates have regions Lorentz Symmetry is broken, including *String Theory*, *Noncommutative Geometry* and *Loop Quantum Gravity*...

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What is CPT Symmetry?

A Simple Definition

CPT is the successive application of *Charge Conjugation*, *Parity* & *Time Reversal* operators.

CPT Theorem

Any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry![*]

Result:

We want to preserve hermiticity and non-local QFT is another story. Hence **CPT Violation** $\Rightarrow$ **Lorentz Violation**.

Caution!

This does *NOT* imply that **Lorentz Violation** $\Rightarrow$ **CPT Violation**.

[*] R. F. Streater & A. S. Wightman (1964) *PCT, Spin and Statistics, and All That*

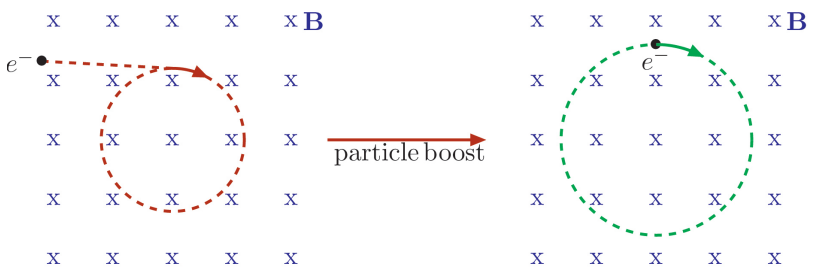
What is Lorentz Symmetry and how do we break it?

- Lorentz transformations → Lorentz Group → Lorentz Symmetry.
- In the context of SR, these are

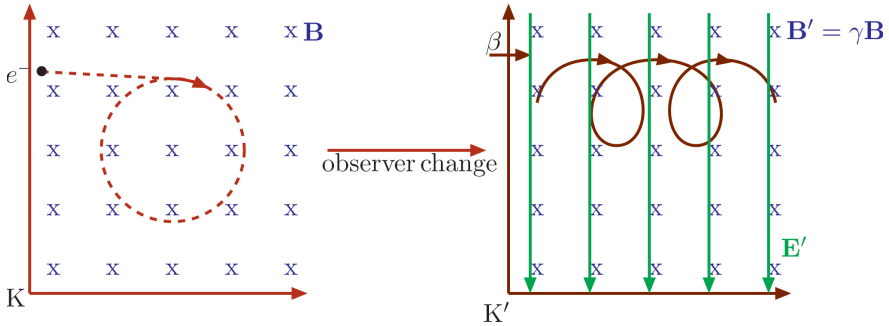
Passive: Particle is unchanged, observer is boosted or rotated.
Active: Observer is unchanged, particle is boosted or rotated.

- Two transformations in QFT:

Observer LT: The same as Passive LT of SR.
Particle LT: The same as Active LT of SR except that the background fields are unchanged.



- Constant background fields transform as scalars in Particle LT and Lorentz Tensors in Observer LT.
- When background fields couple to the usual fields ⟹ Observer LI ✓ Particle LI ✗



Results of the breaking

Result:

- Background field of nonzero expectation value (nonzero expectation value acquired by a Lorentz Tensor) breaks Lorentz invariance $\Rightarrow$ Preffered directions
- There arises free coefficients $\Rightarrow$ to be bounded by experiment!
- Non-zero expectation values are assumed to be low energy effects of a yet-undiscovered theory in Planck scale $\Rightarrow$ **Spontaneous Symmetry Breaking**

Caution!

Observer Lorentz invariance is intact! $\Rightarrow$ Physics is independent of how we set up coordinate system!

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Information about theories out there

Different Philosophies in Lorentz Violation

- Fundamental Theory has *NO* Lorentz Invariance $\Rightarrow$ Observed Lorentz Symmetry is just low energy manifestation
- Explicitly broken Lorentz Group: *Very Special Relativity* by Cohen & Glashow (2006)

Information about theories out there

If exact Lorentz Symmetry of Fundamental Theory is spontaneously broken:

Time Varying Couplings (TVC)

Fundamental couplings are time-varying $\Rightarrow \partial_\mu \alpha$ is preffered direction!

Kinematical Approach: Modifications in transformation laws

- *Robertson-Mansouri-Sexl* Formalism (1949,1977)
- Doubly Special Relativity by (*Kowalski-Glikman*, 2005)

Field Theoretical Models

- Nonstandard optics from quantum space-time (*Gambini & Pullin*, 1999)
- Ultraviolet Modifications of Dispersion Relations in Effective Field Theory (*Myers & Pospelov*, 2003)
- Classification of dimension-5 Lorentz-violating interactions in the standard model by (*Bolokhov & Pospelov*, 2008)

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Philosophy of SME*

- Field Theoretical Model $\Rightarrow$ Terms are added in action level!
- Lorentz violating term $=$ coupling coefficient $\times$ basic fields

Example:

$$\mathcal{L}^{\text{CPT–odd}}_{\text{lepton}} = \frac{1}{2} i (c_L)_{\mu\nu AB} \overline{L}_A \gamma^\mu \overleftrightarrow{D^\nu} L_B + \frac{1}{2} i (c_R)_{\mu\nu AB} \overline{R}_A \gamma^\mu \overleftrightarrow{D^\nu} R_B$$

where $L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L$, $R_A = (l_A)_R$ and c are LV coefficients.

- All such sectors are considered:
  $\mathcal{L}^{\text{CPT–even}}_{\text{lepton}}$, $\mathcal{L}^{\text{CPT–odd}}_{\text{quark}}$, $\mathcal{L}^{\text{CPT–even}}_{\text{quark}}$, $\mathcal{L}_{\text{Higgs}}$, $\mathcal{L}_{\text{Gauge}}$, $\mathcal{L}_{\text{Yukawa}}$, ...

Caution!

NOT all terms of different sectors are independent!

* *Alan Kostelecky*, Phys.Rev.D55:6760-6774,1997

SME has

still conserved quantities:

- Spontaneous symmetry breaking $\Rightarrow$
  - Microcausality ✓
  - Positivity of energy ✓
- Terms are chosen s.t. $\Rightarrow$
  - Observer Lorentz Invariance ✓
  - Energy & Momentum ✓
  - Conventional Quantization ✓
  - Hermiticity ✓
  - Gauge Invariance ✓

and yet alterations:

- Direction dependence $\Rightarrow$ *Anisotropy*
- Wave-packet deformation $\Rightarrow$ *Dispersion*
- Mode splitting $\Rightarrow$ *Birefringence*

Outline

General Form

- Conventional lepton-photon Lagrangian:

$$\mathcal{L}^{\text{QED}}_{\text{lepton-photon}} = \frac{1}{2} i \overline{l_A} \gamma^\mu \overleftrightarrow{D^\nu} l_A - m_A \overline{l_A} l_A - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- CPT Violation Terms:

$$\mathcal{L}^{\text{CPT–odd}}_{\text{lepton}} = -(a_l)_{\mu AB} \overline{l_A} \gamma^\mu l_B - (b_l)_{\mu AB} \overline{l_A} \gamma_5 \gamma^\mu l_B$$

$$\mathcal{L}^{\text{CPT–odd}}_{\text{photon}} = +\frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu}$$

- Sources of tests:
  - Birefringence constraints on cosmological scales
  - Penning Traps
  - Hydrogen and antihydrogen spectroscopy
  - Clock-comparison experiments

Again, some LV terms can be removed via field redefinitions or can be carried into other sectors!

Extended QED with electron, positron & photon only limit!

Most General Lagrangian

$$\mathcal{L}_{\text{Extended QED}} = \mathcal{L}_{\text{Conventional}} + \mathcal{L}^{\text{CPT–odd}}_{\text{electron}} + \mathcal{L}^{\text{CPT–even}}_{\text{electron}} + \mathcal{L}^{\text{CPT–odd}}_{\text{photon}} + \mathcal{L}^{\text{CPT–even}}_{\text{photon}}$$

$$\begin{aligned}\mathcal{L}_{\text{Extended QED}} = & \frac{1}{2} i\overline{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi - m_e \overline{\psi}\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \\ & - a_\mu \overline{\psi}\gamma^\mu \psi - b_\mu \overline{\psi}\gamma_5\gamma^\mu \psi \\ & - \frac{1}{2} H_{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\psi + \frac{1}{2} i c_{\mu\nu}\overline{\psi}\gamma^\mu \overleftrightarrow{D}^\nu \psi + \frac{1}{2} i d_{\mu\nu}\overline{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D}^\nu \psi \\ & + \frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} \\ & - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda}F^{\mu\nu}\end{aligned}$$

Current Bounds on the Parameters

Particle	Coefficient*	Bound†
Photon	k_{AF}	$\sim 10^{-42}$ GeV
Electron	b	$\sim 10^{-22}$ GeV
Electron	c	$\sim 10^{-17}$ GeV
Electron	d	$\sim 10^{-19}$ GeV
Electron	H	$\sim 10^{-26}$ GeV
...		

Result:

$\sim 10^{-42}$ is beyond acceptable range: One needs to consider higher order terms in photon sector.

*Bounds differ within components, only least stringent ones are given.

† *Data Tables for Lorentz and CPT Violation*, Rev.Mod.Phys.83:11,2011

Outline

Philosophy

Assume: Low energy effective description of *Quantum Gravity* is an expansion in energy over a mass scale! Mass scale is probably related to *Planck mass*!

Then:
- Leading order term is *Standard Model*.
- Next-to-leading order term is *minimal SME*.

Caution!

Gravity itself is nonrenormalizable $\Rightarrow$ One expects higher-order terms to be made up of operators of nonrenormalizable dimensions!

- Higher order terms constitute nonrenormalizable SME!

How is it nonrenormalizable?

We know that

It is a necessary *(but not sufficient)* for renormalizable interacting QFT that field operators have mass dimension ≤ 4

Therefore:

The theory is *NOT* renormalizable if there are field operators of higher than four mass dimension!

Caution!

Nonrenormalizable theories are effective theories!

General Form

Most General Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda(\hat{k}_{AF})_\kappa F_{\mu\nu} - \frac{1}{4}F_{\kappa\lambda}(\hat{k}_F)^{\kappa\lambda\mu\nu}F_{\mu\nu}$$

where

$$(\hat{k}_{AF})_\kappa = \sum_{d=\text{odd}} (k_{AF}^{(d)})_\kappa^{\alpha_1\ldots\alpha_{(d-3)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-3)}}$$

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{d=\text{even}} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}}\partial_{\alpha_1}\ldots\partial_{\alpha_{(d-4)}}$$

CPT Violating Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda(\hat{k}_{AF})_\kappa F_{\mu\nu}$$

How should we classify $(\hat{k}_{AF})_\kappa$?

Aim is

To establish a minimum collection of *independent coefficients* associated with operators having physical properties of *direct relevance* to observation and experiment.

Suggestion: Take advantage of role of spatial rotations $\Rightarrow$ SO(3) decomposition!

Advantages:

- *Spin Weighted Spherical Harmonics* are used $\Rightarrow$ Angular momentum eigenstates $\Rightarrow$ Relatively simple transformation rules under rotations!
- Commonly used in astrophysics and well understood.

Caution!

Spatial part of coefficients should be separated according to their spin-weight! We need *Helicity basis*.

How should we classify $(\hat{k}_{AF})_\kappa$?

Definition of Helicity Basis

- Standard angles in spherical coordinates are such that

$$\hat{\boldsymbol{p}} = \sin\theta\cos\phi\,\hat{\boldsymbol{e}}_{\boldsymbol{x}} + \sin\theta\sin\phi\,\hat{\boldsymbol{e}}_{\boldsymbol{y}} + \cos\theta\,\hat{\boldsymbol{e}}_{\boldsymbol{z}}$$

where orthonormal basis vectors satisfy

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} = \hat{\boldsymbol{e}}^{\boldsymbol{r}} = \hat{\boldsymbol{p}} \ , \ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}} = \hat{\boldsymbol{e}}^{\boldsymbol{\theta}} \ , \ \hat{\boldsymbol{e}}_{\boldsymbol{\phi}} = \hat{\boldsymbol{e}}^{\boldsymbol{\phi}}$$

- Complex helicity basis $\{\hat{e}_+, \hat{e}_r, \hat{e}_-\}$ is defined as

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} = \hat{\boldsymbol{e}}^{\boldsymbol{r}} = \hat{\boldsymbol{p}} \ , \ \hat{\boldsymbol{e}}_{\pm} = \hat{\boldsymbol{e}}^{\mp} = \left(\hat{\boldsymbol{e}}_{\boldsymbol{\theta}} \pm i\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}\right)/\sqrt{2}$$

Metric & Levi-Civita in Helicity Basis

$$g_{ab} = g^{ab} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \& \qquad \epsilon_{+r-} = -\epsilon^{+r-} = i$$

How should we classify $(\hat{k}_{AF})_\kappa$?

- Decompose $(\hat{k}_{AF})_\kappa$ in Helicity basis:

$$(\hat{k}_{AF})_\kappa \doteq \begin{pmatrix} (\hat{k}_{AF})_0 \\ (\hat{k}_{AF})_+ \\ (\hat{k}_{AF})_r \\ (\hat{k}_{AF})_- \end{pmatrix}$$

- Expand each component in Spin Weighted Spherical Harmonics:

$$(\hat{k}_{AF})_0 = \sum_{dnjm} \omega^{d-3-n} p^n \, {}_0Y_{jm}(\hat{\boldsymbol{p}}) (k_{AF}^{(d)})_{njm}^{(0B)}$$

$$\ldots = \ldots$$

- New sets of coefficients: $(k_{AF}^{(d)})_{njm}^{(0B)}$, $(k_{AF}^{(d)})_{njm}^{(1B)}$, $(\overset{\neg}{k}_{AF}^{(d)})_{njm}^{(1E)}$

Special Models

1 - Isotropic Models

— In a preferred frame, LV operators preserve rotational invariance!
— One natural choice is *Cosmic Microwave Background*.

2 - Vacuum Models

— LV operators are imposed with $\omega = p$.
— The leading order deviation of the dispersion relation is zero $\Rightarrow \omega \simeq p$

3 - Camouflage Models

LV terms without *leading order birefringence* or *vacuum dispersive effects*.

4 - Vacuum-orthogonal Models

Complementary coefficient subspace to coefficients of *Vacuum Models*!

Outline

Basics

Coefficient Space

According to vacuum properties: $\begin{pmatrix} (k_{AF}^{(d)})_{njm}^{(0B)} \\ (k_{AF}^{(d)})_{njm}^{(1B)} \\ (\overline{k}_{AF}^{(d)})_{njm}^{(1E)} \end{pmatrix} \longrightarrow \begin{pmatrix} k_{(V)jm}^{(d)} \\ (\overline{k}_{AF}^{(d)})_{njm}^{(0B)} \\ (\overline{k}_{AF}^{(d)})_{njm}^{(1B)} \\ (\overline{k}_{AF}^{(d)})_{njm}^{(1E)} \end{pmatrix}$

MARK: Only last three are relevant!

Dispersion Relation in Compact Form

$$0 = (p_\mu p^\mu)^2 - 4\left(p(\hat{k}_{AF})_0 - \omega(\hat{k}_{AF})_r\right)^2 - 8p_\mu p^\mu(\hat{k}_{AF})_+(\hat{k}_{AF})_-$$

Dispersion Relation in Explicit Form

$$
\begin{aligned}
0 = (p_\mu p^\mu)^2 \\
&- 4\Bigg\{ \sum_{dnjm} \omega^{d-3-n} p^n \, {}_0Y_{jm}(\hat{\boldsymbol{p}}) \Bigg\{ (\overset{\neg(d)}{k}_{AF})^{(0B)}_{njm} \frac{p}{\omega^2} \left(\frac{(d-2-n)\omega^2}{d-2-n+j} - \frac{(d-4-n)p^2}{d-4-n+j} \right) \\
&- \frac{dp^2 (\overset{\neg(d)}{k}_{AF})^{(1B)}_{njm}}{(n+4)(n+2)\omega} + (\overset{\neg(d)}{k}_{AF})^{(0B)}_{njm} \frac{j}{p} \left(\frac{\omega^2}{d-2-n+j} - \frac{p^2}{d-4-n+j} \right) + \frac{d\omega}{(n+2)(n+4)} (\overset{\neg(d)}{k}_{AF})^{(1B)}_{njm} \Bigg\} \Bigg\}^2 \\
&+ 8 p_\mu p^\mu \sum_{d_1 d_2 n_1 n_2 j_1 j_2 m_1 m_2} \omega^{d_1+d_2-6-n_1-n_2} p^{n_1+n_2} \, {}_{+1}Y_{j_1 m_1}(\hat{\boldsymbol{p}}) \, {}_{-1}Y_{j_2 m_2}(\hat{\boldsymbol{p}}) \frac{1}{\sqrt{4 j_1 j_2 (j_1+1)(j_2+1)}} \\
&\times \Bigg\{ \Bigg(\left(\frac{\omega j_1 (n_1+1)}{p(d_1-2-n_1+j_1)} - \frac{p j_1 (n_1+3)}{\omega(d_1-4-n_1+j_1)} \right) (\overset{\neg(d_1)}{k}_{AF})^{(0B)}_{n_1 j_1 m_1} + \frac{d_1}{n_1+4} (\overset{\neg(d_1)}{k}_{AF})^{(1B)}_{n_1 j_1 m_1} \Bigg) \\
&\times \Bigg(\left(\frac{\omega j_2 (n_2+1)}{p(d_2-2-n_2+j_2)} - \frac{p j_2 (n_2+3)}{\omega(d_2-4-n_2+j_2)} \right) (\overset{\neg(d_2)}{k}_{AF})^{(0B)}_{n_2 j_2 m_2} + \frac{d_2}{n_2+4} (\overset{\neg(d_2)}{k}_{AF})^{(1B)}_{n_2 j_2 m_2} \Bigg) \\
&+ (\overset{\neg(d_1)}{k}_{AF})^{(1E)}_{n_1 j_1 m_1} (\overset{\neg(d_2)}{k}_{AF})^{(1E)}_{n_2 j_2 m_2} \Bigg\}
\end{aligned}
$$

What to do with this dispersion relation?

Good News!

It can be shown that dispersion relation can be cast into the form

$$0 = (p_\mu p^\mu) \times \Big((p_\mu p^\mu)\mathcal{P}(\omega, p) + \mathcal{R}(\omega, p) \Big)$$

Bad News!

$\mathcal{P}(\omega, p)$ and $\mathcal{R}(\omega, p)$ are quite complicated $\Rightarrow$ Hard to find general nonconventional solutions!

Propagator

General CPT Violating Photon Propagator

$$(\hat{G}^{-1})^{\mu\nu} = -\eta^{\mu\nu}(p_\sigma p^\sigma) + 2i\epsilon^{\mu\kappa\lambda\nu}(\hat{k}_{AF})_\kappa p_\lambda$$

$$\hat{G}_{\mu\nu} \simeq -\frac{1}{p_\sigma p^\sigma}\eta_{\mu\nu} + \frac{2i}{(p_\sigma p^\sigma)^2}\epsilon_{\mu\nu\kappa\lambda}(\hat{k}_{AF})^\kappa p^\lambda$$

Matrix Representation in Explicit Helicity Basis:

$$\hat{G} \doteq \begin{pmatrix} -(p_\sigma p^\sigma) & -2p(\hat{k}_{AF})_- & 0 & 2p(\hat{k}_{AF})_+ \\ -2p(\hat{k}_{AF})_+ & -(p_\sigma p^\sigma) - 2(\omega(\hat{k}_{AF})_r - p(\hat{k}_{AF})_0) & 2\omega(\hat{k}_{AF})_+ & 0 \\ 0 & 2\omega(\hat{k}_{AF})_- & -(p_\sigma p^\sigma) & -2\omega(\hat{k}_{AF})_+ \\ 2p(\hat{k}_{AF})_- & 0 & -2\omega(\hat{k}_{AF})_- & -(p_\sigma p^\sigma) + 2(\omega(\hat{k}_{AF})_r - p(\hat{k}_{AF})_0) \end{pmatrix}^{-1}$$

In No LV Limit:

$$\hat{G} \doteq \begin{pmatrix} -(p_\sigma p^\sigma) & 0 & 0 & 0 \\ 0 & -(p_\sigma p^\sigma) & 0 & 0 \\ 0 & 0 & -(p_\sigma p^\sigma) & 0 \\ 0 & 0 & 0 & -(p_\sigma p^\sigma) \end{pmatrix}^{-1} \Longrightarrow \hat{G}_\mu^{\ \nu} = -\frac{\delta_\mu^{\ \nu}}{(p_\sigma p^\sigma)} \quad \checkmark$$

Polarization Vectors

- Equation to be solved: $M^{\mu\nu} A_\nu = 0$
- Condition for nontrivial solution: $\det(M) = 0$
- $M^{\mu\nu} = -p^\mu p^\nu - 2i\epsilon^{\mu\nu\alpha\beta}(\hat{k}_{AF})_\alpha p_\beta$

General Polarization vector for Vacuum Orthogonal Model

$$\begin{pmatrix} 1 \\ 2\dfrac{(\hat{k}_{AF})_+}{(\hat{k}_{AF})_0 - (\hat{k}_{AF})_r} \\ -1 \\ 2\dfrac{(\hat{k}_{AF})_-}{(\hat{k}_{AF})_0 - (\hat{k}_{AF})_r} \end{pmatrix} \Rightarrow \text{There is only Gauge solution: } \textit{No Physical Solutions}$$

Polarization Vectors

Problem: There is no physical solution for full vacuum orthogonal coefficient space!
Solution: Consider possible constrained coefficient subspaces!

The Only Coefficient Subspace with Physical Solution

- $\left((\hat{k}_{AF})_0 - (\hat{k}_{AF})_r\right) = (\hat{k}_{AF})_+ = (\hat{k}_{AF})_- = 0$
- Resultant polarization vectors are conventional ones!

Mark!

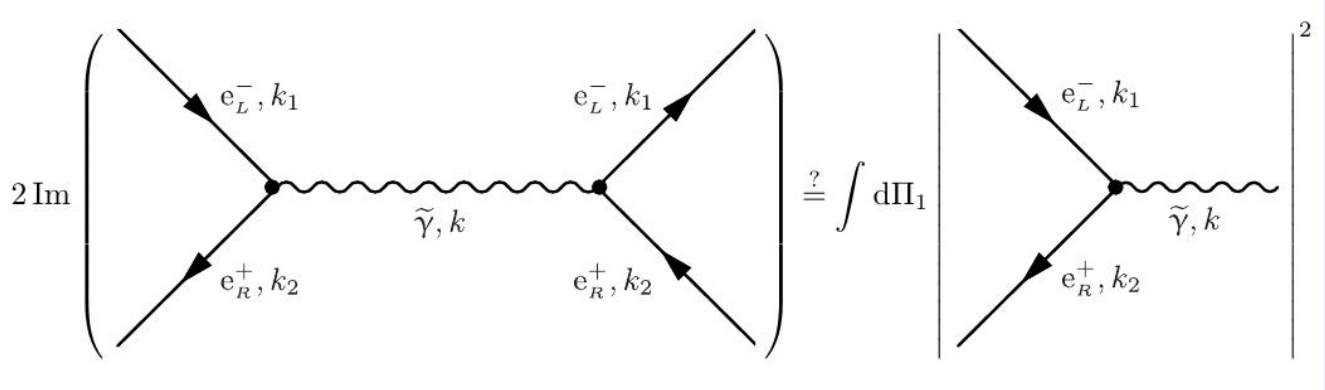
There is no coefficient subspace with one physical solution $\Rightarrow$ No mode-splitting $\Rightarrow$ No vacuum birefringence!

What lies ahead?

The Next Steps for Vacuum Orthogonal Photon Sector

- Calculation of *Gauge Field Commutator* $K^{\mu\nu} = [A^\mu, A^\nu]$
- Analysis of leading order $d = 5$ model in detail
  - Calculation of non-conventional roots of dispersion relation
  - Calculation of non-conventional polarization vectors
  - Check of *Optical Theorem*

Optical Theorem



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* *M. Schreck*, Phys. Rev. D 89, 105019 (2014)

Conclusion

- Low energy effects of many Quantum Theory of Gravity candidates can be searched in *CPT Violating QED*
- *Standard Model Extension* provides the suitable framework for this *Effective Field Theory*
- Experimental bounds eliminates renormalizable photon sector: One needs to consider non-renormalizable photon sector!
- It turns out Vacuum Orthogonal CPT Violating Photon Sector has the dispersion relation

$$0 = (p_\mu p^\mu) \times \Big((p_\mu p^\mu)\mathcal{P}(\omega, p) + \mathcal{R}(\omega, p) \Big)$$

  Vacuum Orthogonal Photon Sector is Vacuum Orthogonal at all orders!
- **There exists possible Lorentz Violations for which conventional polarization vectors are still valid!**

Thank you for your attention!

Any questions?